A Nonblocking Optical Switching Network for Crosstalk-free Permutation[1]

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SUMMARY
Vertical stacking is a novel technique for building switching networks, and packing multiple compatible connections together is an effective strategy to reduce network hardware cost. In this paper, we study the crosstalk-free permutation capability of an optical switching network built on the vertical stacking of optical banyan networks to which packing strategy is applied. We first look into the nonblocking condition of this optical switching network. We then study the crosstalk-free permutation in this network by decomposing a permutation evenly into multiple crosstalk-free partial permutations (CFPPs) and realizing each CFPP in a stacked plane of the network such that a crosstalk-free permutation can be performed in a single pass. We present a rigorous proof of CFPP decomposability of a permutation and also a complete algorithm for CFPP decomposition. The possibility of a tradeoff between the number of passes and the number of planes required for realizing a crosstalk-free permutation in this network is also explored in this paper.
Key words: Vertical stacking, banyan networks, optical crosstalk, directional coupler, optical switching networks.

1.Introduction

It is expected that with the rapid growth of the Internet, the bandwidth demand for data traffic will explode. Optical networks based on wavelength-division multiplexing (WDM) are considered promising to meet this demand. Mesh-based WDM networks have recently attracted a lot of interest, because the Internet topology is meshed in nature and mesh-based WDM networks are more powerful in terms of routing and survivability. Optical switching networks have been widely employed in WDM mesh networks to handle complex mesh topologies and large numbers of wavelengths, particularly at hub locations handling a large amount of traffic. An optical switching network, which is usually composed of basic switching elements (SEs) grouped into switching stages and optical links arranged in some specified interconnection patterns, is expected to have the capability of switching huge optical data at an ultra-high speed.

The most mature technology for implementing the basic 2×2 SEs in optical switching networks is directional-couplers (DCs) [1]. DC is an electro-optical device implemented by manufacturing two waveguides close to each other [1][2][3]. The cross (bar) state of a DC is created by applying a suitable voltage (no voltage) to it. DC can pass multiple-wavelength optical signals, and this makes it ideal for optical cross-connects (OXCs). However, DC suffers from the intrinsic crosstalk problem [1][4]. When two optical signals pass through a DC, a portion of optical power in one waveguide will be coupled into the other unintended waveguide, this undesirable coupling is called the first-order crosstalk to the unintended channel of waveguide. This first-order SE crosstalk will propagate downstream stage by stage, introducing high order crosstalks at a reduced magnitude. Although signal attenuation is another drawback of DC-based optical switching networks, it can be overcome by optical amplifiers.

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However, optical amplifiers are usually linear, in the sense that they amplify signals as well as crosstalk. Because of the stringent bit-error rate requirement of optical transmission facilities, elimination of crosstalk in a DC-based switching network has become an important issue for making optical networks work properly [4][5][6][7][8][9][10][11]. By ensuring that only one signal at a time passes through a DC, the first-order crosstalk can be eliminated, which provides a cost-effective solution to the crosstalk problem.

The topology of banyan type networks [12][13][14] [15] is a very popular structure for building switching networks. This class of networks is characterized by having a unique path from each network input to each network output, a small depth, and a simple switch setting ability (self-routing). These characteristics have made banyan topology promising for constructing DC-based optical switching networks, because loss and attenuation of an optical signal are proportional to the number of couplers that the optical signal passes through. The unique path property of banyan networks makes them blocking networks. Vertical stacking of multiple copies of an optical banyan network is a novel scheme for constructing a nonblocking optical switching network [16]. Fig.1 illustrates the vertical stacking scheme. The resulting networks, vertically stacked optical banyan (VSOB) networks, neither increase the number of stages nor sacrifice the self-routing property of banyan networks.

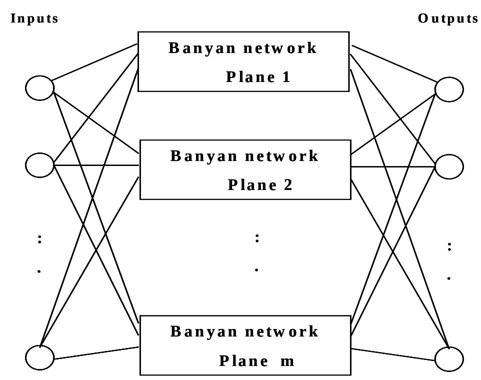


Fig.1. Vertical stacking of banyan networks

In this paper, we will focus on the VSOB networks that are free of first-order crosstalk in SEs (hereafter referred to as crosstalk-free). It is the crosstalk-free constraint that makes the analysis of optical switching networks different from that of electronic ones. Conventionally, blocking happens when two connections intend to use the same link, which we refer to as link-blocking. In VSOB networks, however, there is another type of blocking. Sometimes, all the links along the path of a new connection are available, but adding the new connection will cause some paths, including the new one, to violate the crosstalk-free constraint. In this case, the connection cannot be added even if the path is available. We refer to this second type of blocking as crosstalk-blocking. Obviously, all signals passing through a VSOB network should follow node-disjoint paths in transmission to avoid crosstalk-blocking.

A switching network is strictly nonblocking if every input has a dedicated path through the network to each output, and thus any input signal can be routed to any unused output regardless of the way other input signals are routed. A network is wide-sense nonblocking if any input can be routed to any unused output through a non-dedicated path in the network, and a specific routing algorithm must be followed to guarantee that all of the desired connections can be established [17][18]. Available results indicate that, under the constraint of crosstalk-free, hardware cost is prohibitively high for a strictly nonblocking VSOB network [5] or a wide-sense nonblocking VSOB network [11]. Packing strategy is a commonly used routing control strategy [19], and it is generally believed that packing can improve network performance and reduce network cost [20][21]. Under packing strategy, a connection is realized on a path found by trying the most used part of the network first and the least used part last. In this paper, we will first look into the nonblocking condition for a VSOB network under the packing strategy. We then study the crosstalk-free permutation in a VSOB network by decomposing a permutation evenly into multiple crosstalk-free partial permutations (CFPPs) and realizing each CFPP in a stacked copy of the VSOB network. The main contributions of our work are the following:

- We prove that the nonblocking condition of a VSOB network under packing strategy is the same as the nonblocking condition of a rearrangeable VSOB network given in [7], and this nonblocking VSOB network under packing strategy is optimal in the sense that it consists of the minimum number of planes required by a nonblocking VSOB network.
- We provide a rigorous proof of CFPP decomposability by using a combinatorial theorem of P.Hall [22] and also a complete CFPP decomposition algorithm based on the *Euler Split* technique for edge coloring of bipartite graphs [23].
- We explore the possible tradeoff between number of passes and number of planes required to realize a crosstalk-free permutation in a VSOB network.

The rest of the paper is organized as follows: Section 2 studies the nonblocking condition of a VSOB network applying packing strategy. Section 3 provides the proof of CFPP decomposability and also a complete algorithm for CFPP decomposition. Section 4 discusses the possibility of a tradeoff between the number of planes and the number of passes for realizing crosstalk-free permutations in a VSOB network, and Section 5 summarizes the contributions of this paper.

2. Noblocking VSOB network under packing strategy

First, we need to introduce some notations and definitions. A typical $N\times N$ banyan network consists of $\log_2 N$ stages, each stage contains $N/2$ 2×2 switches and the link connections between adjacent stages are implemented by recursively applying the butterfly interconnection pattern, as shown in Fig.2.

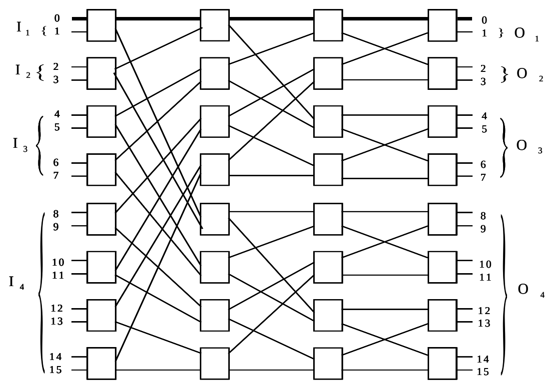


Fig.2 16×16 banyan network (even number of stages)

All paths of a banyan network have the same property in terms of blocking. To study the blocking property, we can arbitrarily select an input and an output in a banyan network and set up a connection between them. Throughout this paper, we will select the path between the first input and the first output and try to set up a connection between them. We call the path between the input-output pair the tagged path. The links and the SEs along the path are called the tagged links and the tagged SEs, respectively. The stages of SE are numbered from left (stage 1) to right (stage $\log_2 N$). For a tagged path, an input intersecting set I_i associated with stage i ($\forall 1\le i \le \log_2 N$) is defined as the set of all inputs that intersect a tagged SE at stage i. Likewise, an output intersecting set O_i associated with stage i is the set of all outputs that intersect a tagged SE at stage $\log_2 N - i + 1$.

For convenience, we use VSOB(N,m) to denote an $N\times N$ VSOB network that has m stacked copies (planes) of an $N\times N$ banyan network. Then we have the following theorem regarding the nonblocking condition of a VSOB(N,m) network applying packing strategy.

Theorem1: Under packing strategy, a VSOB(N,m) network is nonblocking if and only if

$$m = \begin{cases} \sqrt{N}, & \text{if } \log_2 N \text{ even} \\ \sqrt{2N}, & \text{if } \log_2 N \text{ odd.} \end{cases} \tag{1}$$

Proof Under the constraint of crosstalk-free, only one light signal is allowed to pass through an SE at a time. Whenever one tagged SE is used by a connection, the path is blocked.

We first examine the case in which $\log_2 N$ is odd and use the 32×32 banyan network in Fig.3 for illustration.

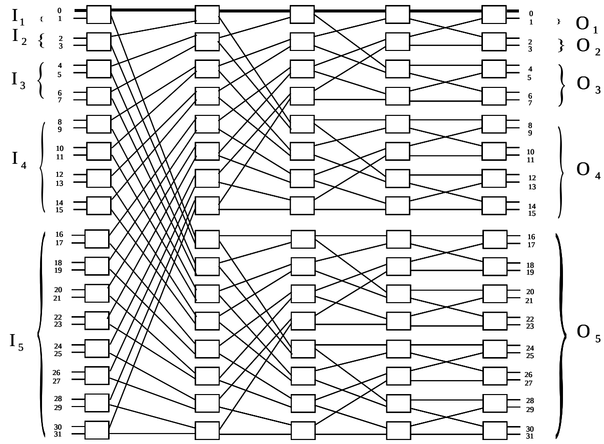


Fig.3 32×32 banyan network (odd number of stages)

For the tagged path (between the first input and the first output), the maximum number of conflicts is determined by the number of inputs from set $\bigcup_{i=1}^{(1/2)(\log_2 N+1)} I_i$ and the number of outputs from set $\bigcup_{i=1}^{(1/2)(\log_2 N+1)} O_i$. The worst-case condition is when every input in set $\bigcup_{i=1}^{(1/2)(\log_2 N+1)} I_i$ and every output in set $\bigcup_{i=1}^{(1/2)(\log_2 N+1)} O_i$ generates a connection to intersect the tagged path. We would like to find the maximum number of planes that can be blocked under this worst-case condition when packing strategy is applied. To block the tagged path, connections originating from set $I_{(1/2)(\log_2 N+1)}$ must be destined for set $\bigcup_{i=1}^{(1/2)(\log_2 N+1)} O_i$; these connections all share the tagged SE in the middle stage (1/2)($\log_2 N$ +1) and must fall within $\left|I_{(1/2)(\log_2 N+1)}\right| = \sqrt{N/2}$ separate planes. However, connections that originate from set $\bigcup_{i=1}^{(1/2)(\log_2 N-1)} I_i$ and block the tagged path may or may not be destined for set $\bigcup_{i=1}^{(1/2)(\log_2 N+1)} O_i$. Since $\sum_{i=1}^{(1/2)(\log_2 N-1)} |I_i| = \sqrt{N/2}-1$, we suppose that there are l ($0 \le l \le \sqrt{N/2}-1$) such connections from $\bigcup_{i=1}^{(1/2)(\log_2 N-1)} I_i$ are destined for set $\bigcup_{i=1}^{(1/2)(\log_2 N+1)} O_i$ and the other $\sqrt{N/2}-1-l$ connections are not destined for set $\bigcup_{i=1}^{(1/2)(\log_2 N+1)} O_i$. These l connections from $\bigcup_{i=1}^{(1/2)(\log_2 N-1)} I_i$ will share the tagged SE in the middle stage (1/2)($\log_2 N$ +1), so they must fall within l separate planes other than the $\sqrt{N/2}$ planes devoted to the connections from set $I_{(1/2)(\log_2 N+1)}$. The other $\sqrt{N/2}-1- l$ connections from $\bigcup_{i=1}^{(1/2)(\log_2 N-1)} I_i$, however, will never share any SEs (including tagged SEs) with the connections from set $I_{(1/2)(\log_2 N+1)}$ to set $\bigcup_{i=1}^{(1/2)(\log_2 N+1)} O_i$. Under the packing strategy, in which a connection is realized on a path found by trying the most used plane first and the least used plane last, these $\sqrt{N/2}-1- l$ connections from $\bigcup_{i=1}^{(1/2)(\log_2 N-1)} I_i$ can share these $\sqrt{N/2}$ planes devoted to the connections from $I_{(1/2)(\log_2 N+1)}$ to $\bigcup_{i=1}^{(1/2)(\log_2 N+1)} O_i$. Therefore, the total number of blocked planes under this scenario will be $\sqrt{N/2}+l$ if packing strategy is applied. Note that $0 \le l \le \sqrt{N/2}-1$, so the maximum number of blocked planes is

$\sqrt{N/2}+\sqrt{N/2}-1=\sqrt{2N}-1$, which corresponds to the scenario that all connections originating from $\bigcup_{i=1}^{(1/2)(\log_2 N+1)} I_i$ are destined for set $\bigcup_{i=1}^{(1/2)(\log_2 N+1)} O_i$. The tagged connection must be established in an extra plane, which brings the total number of planes to $\sqrt{2N}$.

When $\log_2 N$ is even (Fig.2), the discussion is similar. The maximum number of conflicts with the tagged path is determined by the number of inputs from set $\bigcup_{i=1}^{(1/2)\log_2 N} I_i$ and the number of outputs from set $\bigcup_{i=1}^{(1/2)\log_2 N} O_i$, and the worst-case condition is when every input in set $\bigcup_{i=1}^{(1/2)\log_2 N} I_i$ and every output in set $\bigcup_{i=1}^{(1/2)\log_2 N} O_i$ generates a connection to intersect the tagged path. To block the tagged path, the connections from $\bigcup_{i=1}^{(1/2)\log_2 N} I_i$ may or may not be destined for the outputs in set $\bigcup_{i=1}^{(1/2)\log_2 N} O_i$ and the connections destined for the outputs in set $\bigcup_{i=1}^{(1/2)\log_2 N} O_i$ may or may not originate from set $\bigcup_{i=1}^{(1/2)\log_2 N} I_i$. Since $\sum_{i=1}^{(1/2)\log_2 N}|I_i| = \sum_{i=1}^{(1/2)\log_2 N}|O_i| = \sqrt{N}-1$, we assume that there are l ($0\le l \le \sqrt{N}-1$) connections from $\bigcup_{i=1}^{(1/2)\log_2 N} I_i$ to $\bigcup_{i=1}^{(1/2)\log_2 N} O_i$. These l connections will block l planes, because they all share the tagged SEs in stages $(1/2)(\log_2 N)$ and $(1/2)(\log_2 N+2)$. By the symmetry of banyan networks, there should be two groups of other connections from $\bigcup_{i=1}^{(1/2)\log_2 N} I_i$ to $\bigcup_{i=1}^{(1/2)\log_2 N} O_{\log_2 N-i+1}$ and from $\bigcup_{i=1}^{(1/2)\log_2 N} I_{\log_2 N-i+1}$ to $\bigcup_{i=1}^{(1/2)\log_2 N} O_i$ respectively, $\sqrt{N}-1-l$ connections per group. Since these two groups of connections do not share any SE (including tagged SEs), by applying the packing strategy, a connection from one group can share a plane with a connection from another group, and the connections from these two groups can block at most $\sqrt{N}-1-l$ planes. Therefore, the maximum number of planes that can be blocked by the connections that originate from set $\bigcup_{i=1}^{(1/2)\log_2 N} I_i$ and connections that are destined for set $\bigcup_{i=1}^{(1/2)\log_2 N} O_i$ will be $\left(\sqrt{N}-1-l\right)+l=\sqrt{N}-1$; one scenario corresponding to this maximum number of blocked planes is that all connections that originate from set $\bigcup_{i=1}^{(1/2)\log_2 N} I_i$ are destined for set $\bigcup_{i=1}^{(1/2)\log_2 N} O_i$. Adding this number up with an extra plane needed to carry the tagged connection, we determine that the maximum number of planes required is $\left(\sqrt{N}-1\right)+1=\sqrt{N}$.

The above analysis indicates that, under the packing strategy, $\sqrt{N}$ ($\sqrt{2N}$) planes are sufficient for a VSOB(N,m) to be nonblocking if $\log_2 N$ is even (odd). It is easy to see that this condition is also necessary because we need at least $\sqrt{N}$ ($\sqrt{2N}$) planes to realize the crosstalk-free identity permutation in a VSOB(N,m) network when $\log_2 N$ is even (odd).
QED.

It is interesting to note that the nonblocking condition of a VSOB network under packing strategy is the same as the nonblocking condition of a rearrangeable VSOB network given in [7] and that the hardware cost of this network is much lower than the cost of its strictly nonblocking counterpart [5] and its wide-sense nonblocking counterpart [11]. Note that under the constraint of crosstalk-free, we need at least $2^{\lfloor(\log_2 N+1)/2\rfloor}$ planes to realize the identity permutation in a VSOB(N,m) network, no mater what kind of routing algorithm is used. Thus, $2^{\lfloor(\log_2 N+1)/2\rfloor}$ is the lower bound on the number of planes required by a nonblocking VSOB(N,m) network. In this sense, the nonblocking VSOB network under packing strategy is optimal, because it consists of the minimum number of planes required by a nonblocking VSOB network.

Hereafter, we will use VSOB (N) to refer to the VSOB(N,m) consisting of $2^{\lfloor(\log_2 N+1)/2\rfloor}$ planes.

3.Crosstalk-free permutation in a VSOB(N) network

The above result indicates that all permutations are crosstalk-free realizable in a VSOB(N) network that consists of $2^{\lfloor(\log_2 N+1)/2\rfloor}$ planes. In this section, we will give a complete algorithm for realizing a crosstalk-free permutation in a VSOB(N) network. The basic idea of realizing a crosstalk-free permutation in such a network is to decompose it evenly into $2^{\lfloor(\log_2 N+1)/2\rfloor}$ partial permutations, each of which is crosstalk-free realizable in a single plane of the vertically stacked network.

3.1 Crosstalk-Free Partial Permutation (CFPP)

It is usually desirable to distribute connection requests evenly in a network, so that a good load balance can be kept. To realize a crosstalk-free permutation in a VSOB(N) network with a good load balance, we wish to decompose the permutation evenly into $2^{\lfloor(\log_2 N+1)/2\rfloor}$ partial permutations and realize each of them crosstalk-free in a single plane of the network. For convenience, we denote $2^{\lfloor(\log_2 N+1)/2\rfloor}$ as T and introduce the following definition.

Definition 1 A partial permutation $P = \begin{pmatrix} x_0, x_1, \cdots, x_{N/T-1} \\ y_0, y_1, \cdots, y_{N/T-1} \end{pmatrix}$ for a VSOB(N) network, where input x_i is mapped to output y_i, with x_i, y_i $\in\{0,1,\ldots,N\text{-}1\}$ and $x_0 < x_1 < \cdots < x_{N/T-1}$, is referred to as a crosstalk-free partial permutation (CFPP) of the network if P is crosstalk-free realizable in one of the network's planes.

Example 1 The decomposition of a permutation into CFPPs.

$$\begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 \\ 8 & 2 & 0 & 12 & 4 & 13 & 3 & 11 & 9 & 1 & 6 & 7 & 5 & 10 & 15 & 14 \end{pmatrix} \Rightarrow$$

$$\begin{pmatrix} 1 & 7 & 10 & 14 \\ 2 & 11 & 6 & 15 \end{pmatrix} \circ \begin{pmatrix} 2 & 5 & 8 & 12 \\ 0 & 13 & 9 & 5 \end{pmatrix} \circ \begin{pmatrix} 0 & 6 & 11 & 15 \\ 8 & 3 & 7 & 14 \end{pmatrix} \circ \begin{pmatrix} 3 & 4 & 9 & 13 \\ 12 & 4 & 1 & 10 \end{pmatrix}$$

3.2 CFPP decomposability of a permutation

Theorem 1 indicates that any permutation is crosstalk-free realizable in a VSOB(N) network but does not guarantee that any permutation can be evenly decomposed into $2^{\lfloor(\log_2 N+1)/2\rfloor}$ CFPPs. We will prove in this section that we can actually decompose any permutation evenly into $2^{\lfloor(\log_2 N+1)/2\rfloor}$ CFPPs for a VSOB(N) network; then in the next section we will provide a complete algorithm for such a decomposition.

The CFPP decomposability of a permutation will be presented in Theorem 2. We first need to establish the following lemma that will be used to prove Theorem 2.

For an $N \times N$ banyan network and a given integer i ($0\le i \le(1/2)(\log_2 N\text{-}2)$ when $\log_2 N$ is even, $0\le i \le (1/2)(\log_2 N\text{-}1)$ when $\log_2 N$ is odd), we group both input switches and output switches of the network into sets:

$$I_j^{[i]} = \{u_{2^i\cdot j}, u_{2^i\cdot j+1}, \cdots, u_{2^i\cdot j+2^i-1}\},\ O_j^{[i]} = \{v_{2^i\cdot j}, v_{2^i\cdot j+1}, \cdots, v_{2^i\cdot j+2^i-1}\},\ 0 \le j \le \tfrac{N}{2^{i+1}} - 1 \qquad (2)$$

where u_0, u_1,…, $u_{N/2\text{-}1}$ are the N/2 inputs switches and v_0, v_1 ,…,$v_{N/2\text{-}1}$ are the N/2 outputs switches. Then we have the following result regarding the crosstalk-free property of a banyan network.

Lemma 1 For the two inputs (outputs) of any two one-pair mappings in an $N \times N$ banyan network, if their corresponding input switches (output switches) belong to two different sets of input (output) switches defined in (2), the two mappings will be crosstalk-free in the first (last) i+1 stages of the network.

Proof. For an $N \times N$ banyan network and a given integer i (here $1\le i \le(1/2)(\log_2 N\text{-}2)$ if $\log_2 N$ is even, $1\le i \le (1/2)(\log_2 N\text{-}1)$ if $\log_2 N$ is odd), let $\begin{pmatrix} x_{j_1} \\ y_{k_1} \end{pmatrix}$ and $\begin{pmatrix} x_{j_2} \\ y_{k_2} \end{pmatrix}$ be two one-pair mappings the network with input switch corresponding to x_{j_1} belongs to $I_{j_1}^{[i]} = \{u_{2^i\cdot j_1}, u_{2^i\cdot j_1+1}, \cdots, u_{2^i\cdot j_1+2^i-1}\}$ and input switch corresponding to x_{j_2} belongs to

$I_{j_2}^{[i]} = \{u_{2^i \cdot j_2}, u_{2^i \cdot j_2+1}, \cdots, u_{2^i \cdot j_2 + 2^i - 1}\}$, here $0 \le j_1, j_2 \le \frac{N}{2^{i+1}} - 1$. By the definition of banyan network, x_{j_1} can only pass through the switches belonging to $A_{j_1}^{[i,h]} = \bigcup_{k=0}^{2^{h-1}-1} \left\{ u^{[h]}_{2^{i+1-h} \cdot j_1 + k \cdot \frac{N}{2^h}}, u^{[h]}_{2^{i+1-h} \cdot j_1 + k \cdot \frac{N}{2^h} + 1}, \cdots, u^{[h]}_{2^{i+1-h} \cdot j_1 + k \cdot \frac{N}{2^h} + 2^{i+1-h} - 1} \right\}$ and x_{j_2} can only pass through the switches belonging to $A_{j_2}^{[i,h]} = \bigcup_{k=0}^{2^{h-1}-1} \left\{ u^{[h]}_{2^{i+1-h} \cdot j_2 + k \cdot \frac{N}{2^h}}, u^{[h]}_{2^{i+1-h} \cdot j_2 + k \cdot \frac{N}{2^h} + 1}, \cdots, u^{[h]}_{2^{i+1-h} \cdot j_2 + k \cdot \frac{N}{2^h} + 2^{i+1-h} - 1} \right\}$ in the *h-th* stage of the network for $1 \le h \le i+1$,here $u_0^{[h]}, u_1^{[h]}, \cdots, u_{N/2-1}^{[h]}$ are the $N/2$ switches in the *h-th* stage and $u_m^{[1]} = u_m$ for $0 \le m \le N/2 - 1$. If $j_1 \ne j_2$, $A_{j_1}^{[i,h]}$ will be disjoint with $A_{j_2}^{[i,h]}$, so x_{j_1} and x_{j_2} will not share any switch in the *h-th* stage of the network for $1 \le h \le i+1$, thus, x_{j_1} and x_{j_2} will be crosstalk-free in the *h-th* stage of the network for $1 \le h \le i+1$ if $j_1 \ne j_2$. Following an argument similar to that for the input sets, we also can prove that if the two output switches corresponding to two outputs of any two one-pair mappings in the network belong to two different output sets defined above, the two mappings will be crosstalk-free in the last $i+1$ stages of the network.
QED.

We are now in the position to prove the following theorem regarding the CFPP decomposability of a permutation.

Theorem 2: Any permutation of an N-element set $\{0,1,\ldots, N-1\}$ can be decomposed into $\sqrt{N}$ ($\sqrt{2N}$) CFPPs if $\log_2 N$ is even (odd).
Proof We will prove the theorem based on P.Hall's distinct system representative theorem [22] and the treatments established in [24].

Let the permutation be the form

$$\begin{pmatrix} x_0, x_1, \cdots, x_{N-1} \\ y_0, y_1, \cdots, y_{N-1} \end{pmatrix} \tag{3}$$

where input $x_i = i$ is mapped to output y_i for $0 \le i \le N-1$, and $\{y_0, y_1, \ldots, y_{N-1}\} = \{0,1,\ldots,N-1\}$.

(1) When $\log_2 N$ is even, we decompose the permutation into $\sqrt{N}$ partial permutations, each of which has $\sqrt{N}$ elements

$$\begin{pmatrix} x_0 \cdots x_{\sqrt{N}-1} \\ y_0 \cdots y_{\sqrt{N}-1} \end{pmatrix}, \begin{pmatrix} x_{\sqrt{N}} \cdots x_{\sqrt{N}+\sqrt{N}-1} \\ y_{\sqrt{N}} \cdots y_{\sqrt{N}+\sqrt{N}-1} \end{pmatrix}, \ldots, \begin{pmatrix} x_{(\sqrt{N}-1)\sqrt{N}} \cdots x_{N-1} \\ y_{(\sqrt{N}-1)\sqrt{N}} \cdots y_{N-1} \end{pmatrix} \tag{4}$$

Note that for the j^{th} partial permutation $\begin{pmatrix} x_{j\cdot\sqrt{N}} \cdots x_{j\cdot\sqrt{N}+\sqrt{N}-1} \\ y_{j\cdot\sqrt{N}} \cdots y_{j\cdot\sqrt{N}+\sqrt{N}-1} \end{pmatrix}$ ($0 \le j \le \sqrt{N} - 1$), we have:

$$\begin{pmatrix} \left\lfloor \frac{x_{j\cdot\sqrt{N}}}{\sqrt{N}} \right\rfloor \cdots \left\lfloor \frac{x_{j\cdot\sqrt{N}+\sqrt{N}-1}}{\sqrt{N}} \right\rfloor \\ \left\lfloor \frac{y_{j\cdot\sqrt{N}}}{\sqrt{N}} \right\rfloor \cdots \left\lfloor \frac{y_{j\cdot\sqrt{N}+\sqrt{N}-1}}{\sqrt{N}} \right\rfloor \end{pmatrix} = \begin{pmatrix} j & \cdots & j \\ \left\lfloor \frac{y_{j\cdot\sqrt{N}}}{\sqrt{N}} \right\rfloor & \cdots & \left\lfloor \frac{y_{j\cdot\sqrt{N}+\sqrt{N}-1}}{\sqrt{N}} \right\rfloor \end{pmatrix} \tag{5}$$

Denote $\left\{ \left\lfloor \frac{y_{j\cdot\sqrt{N}}}{\sqrt{N}} \right\rfloor, \left\lfloor \frac{y_{j\cdot\sqrt{N}+1}}{\sqrt{N}} \right\rfloor, \ldots, \left\lfloor \frac{y_{j\cdot\sqrt{N}+\sqrt{N}-1}}{\sqrt{N}} \right\rfloor \right\}$ as B_j for $0 \le j \le \sqrt{N} - 1$. It is easy to see from $\{y_0, y_1, \ldots, y_{N-1}\} = \{0,1,\ldots,N-1\}$ that there are $\sqrt{N}$ 0's, $\sqrt{N}$ 1's, … , and $\sqrt{N}$ ($\sqrt{N}$ -1)'s distributed in sets $B_0, B_1, \ldots, B_{\sqrt{N}-1}$. Thus, for any k sets ($1 \le k \le \sqrt{N}$), $B_{i_1}, B_{i_2}, \ldots, B_{i_k}$,there are a total of $\sqrt{N} \cdot k$ elements that form a multiset with the multiplicity of each element no more than $\sqrt{N}$. Therefore, the cardinality of the union of these sets satisfies:

$$\left| B_{i_1} \cup B_{i_2} \cup \ldots \cup B_{i_k} \right| \ge k \tag{6}$$

By P.Hall's distinct system representatives theorem [22], we know that (6) is the necessary and sufficient condition for these $\sqrt{N}$ sets $B_0, B_1, \ldots, B_{\sqrt{N}-1}$ to have a set of distinct system representatives, so there exist $b_0 \in B_0$, $b_1 \in B_1, \ldots, b_{\sqrt{N}-1} \in B_{\sqrt{N}-1}$ such that $b_i \neq b_j$ for any $i \neq j$ ($0 \le i,j \le \sqrt{N}$ -1). This indicate that

$$\{b_0, b_1, \ldots, b_{\sqrt{N}-1}\} = \{0,1,\ldots,\sqrt{N}-1\} \tag{7}$$

Since $b_j \in B_j = \left\{ \left\lfloor \frac{y_{j\cdot\sqrt{N}}}{\sqrt{N}} \right\rfloor, \left\lfloor \frac{y_{j\cdot\sqrt{N}+1}}{\sqrt{N}} \right\rfloor, \ldots, \left\lfloor \frac{y_{j\cdot\sqrt{N}+\sqrt{N}-1}}{\sqrt{N}} \right\rfloor \right\}$, we denote b_j as $\left\lfloor \frac{y_{j\cdot\sqrt{N}+d_j}}{\sqrt{N}} \right\rfloor$. Then we can get the following partial permutation based on the selection of b_j ($0 \le j \le \sqrt{N}$ -1).

$$\begin{pmatrix} x_{d_0} & x_{\sqrt{N}+d_1} & x_{2\sqrt{N}+d_2} & \cdots & x_{(\sqrt{N}-1)\sqrt{N}+d_{(\sqrt{N}-1)}} \\ y_{d_0} & y_{\sqrt{N}+d_1} & y_{2\sqrt{N}+d_2} & \cdots & y_{(\sqrt{N}-1)\sqrt{N}+d_{(\sqrt{N}-1)}} \end{pmatrix} \tag{8}$$

For subscript d_j, it easy to see that

$$0 \le d_j \le \sqrt{N}-1,\ j = 0,1,\cdots,\sqrt{N}-1. \tag{9}$$

We will show that (8) is a CFPP. Note that

$$\begin{aligned} &\left\{ \left\lfloor \frac{x_{d_0}}{\sqrt{N}} \right\rfloor, \left\lfloor \frac{x_{\sqrt{N}+d_1}}{\sqrt{N}} \right\rfloor, \left\lfloor \frac{x_{2\sqrt{N}+d_2}}{\sqrt{N}} \right\rfloor, \left\lfloor \frac{x_{(\sqrt{N}-1)\sqrt{N}+d_{\sqrt{N}-1}}}{\sqrt{N}} \right\rfloor \right\} = \\ &= \left\{ \left\lfloor \frac{d_0}{\sqrt{N}} \right\rfloor, \left\lfloor \frac{\sqrt{N}+d_1}{\sqrt{N}} \right\rfloor, \left\lfloor \frac{2\sqrt{N}+d_2}{\sqrt{N}} \right\rfloor, \left\lfloor \frac{(\sqrt{N}-1)\cdot\sqrt{N}+d_{\sqrt{N}-1}}{\sqrt{N}} \right\rfloor \right\} \overset{\text{by (9)}}{=} \{0,1,\ldots,\sqrt{N}-1\} \\ &\left\{ \left\lfloor \frac{y_{d_0}}{\sqrt{N}} \right\rfloor, \left\lfloor \frac{y_{\sqrt{N}+d_1}}{\sqrt{N}} \right\rfloor, \left\lfloor \frac{y_{2\sqrt{N}+d_2}}{\sqrt{N}} \right\rfloor, \left\lfloor \frac{y_{(\sqrt{N}-1)\sqrt{N}+d_{\sqrt{N}-1}}}{\sqrt{N}} \right\rfloor \right\} = \\ &= \{b_0, b_1, b_2, \ldots, b_{\sqrt{N}-1}\} \overset{\text{by (7)}}{=} \{0,1,\ldots,\sqrt{N}-1\} \end{aligned} \tag{10}$$

Therefore, by Definition 1 and Lemma 1, partial permutation (8) is a CFPP, and we can take it as the first CFPP decomposed from the permutation.

After we delete the inputs and outputs corresponding the first CFPP from the $\sqrt{N}$ $\sqrt{N}$ -element partial permutations in (4), we can get $\sqrt{N}$ ($\sqrt{N}$ -1)-element partial permutations, and the decomposition process above can then be applied to the $\sqrt{N}$ ($\sqrt{N}$ -1)-element partial permutations to get the second CFPP. By applying the process above recursively, we can finally decompose the original permutation into $\sqrt{N}$ CFPPs.

(2) When $\log_2 N$ is odd, we decompose the permutation into $\sqrt{N/2}$ partial permutations, each of which has $\sqrt{2N}$ elements

$$\begin{pmatrix} x_0 \cdots x_{\sqrt{2N}-1} \\ y_0 \cdots y_{\sqrt{2N}-1} \end{pmatrix},\ \begin{pmatrix} x_{\sqrt{2N}} \cdots x_{\sqrt{2N}+\sqrt{2N}-1} \\ y_{\sqrt{2N}} \cdots y_{\sqrt{2N}+\sqrt{2N}-1} \end{pmatrix},\ \ldots,\ \begin{pmatrix} x_{(\sqrt{N/2}-1)\sqrt{2N}} \cdots x_{N-1} \\ y_{(\sqrt{N/2}-1)\sqrt{2N}} \cdots y_{N-1} \end{pmatrix}$$

Following the similar arguments used above, we can prove that the permutation can be decomposed into $\sqrt{2N}$ CFPPs when $\log_2 N$ is odd.

QED.

3.3 CFPP decomposition algorithm

Theorem 2 guarantees the CFPP decomposability of a permutation. In this section, we will give a complete decomposition algorithm to decompose any permutation of an N-element set {0,1,…, N-1} into $\sqrt{N}$ ($\sqrt{2N}$) CFPPs if $\log_2 N$ is even (odd).

Given a permutation of form (3), in which input $x_i = i$ is mapped to output y_i for $0 \leq i \leq N$-1, and $\{y_0, y_1, \ldots, y_{N-1}\} = \{0, 1, \ldots, N-1\}$, we define $I_j = \{u_j\}$ and $O_j = \{v_j\}$ for $0 \leq j \leq \frac{N}{2} - 1$, and u_0, u_1 ,…, $u_{N/2-1}$ are the N/2 inputs switches and v_0, v_1 ,…,$v_{N/2-1}$ are the N/2 outputs switches. Now we construct an undirected bipartite graph $G = (V_1, V_2 ; E)$. The vertex sets of G are defined as

$$V_1 = \left\{I_0, I_1, \cdots, I_{\frac{N}{2}-1}\right\},\ V_2 = \left\{O_0, O_1, \cdots, O_{\frac{N}{2}-1}\right\}$$

The edge set E is defined as: for any one-pair mapping $\begin{pmatrix} x_i \\ y_i \end{pmatrix}$ in the permutation, if the input switch corresponding to x_i belongs to I_{j_1} and the output switch corresponding to y_i belongs to O_{j_2}, then there is an edge between vertex I_{j_1} and vertex O_{j_2} in E. We also assign each edge in E a label representing the corresponding one-pair mapping in the permutation. Since each vertex of G has an even degree (degree 2), then the bipartite graph only consists of closed paths (connected component), an *Euler split* [23] of bipartite graph G splits G into two bipartite graphs $G_1 = (V_1, V_2 ; E_1)$ and $G_2 = (V_1, V_2 ; E_2)$ where E_1 and E_2 are formed by scanning the paths of each component of G and alternately placing one edge into E_1 and one edge into E_2 . In particular, in our case that each edge vertex has degree two, the closed paths become circle and the Euler split of G can be performed in a simple way as follows:

Euler Split Algorithm:

1. Construct a bipartite graph G for a given permutation.
2. For each connected component of G, start from a vertex of this component in V_1, traverse through an unvisited edge to the neighbor vertex in V_2, back and forth until returning to the starting vertex.
3. During the traversing, a visited edge will be placed into E_1 if the traverse direction on this edge is from V_1 to V_2 and will be placed into E_2 if the traverse direction is opposite.

The above split is correct because, from graph theory [25], we know that for a component of a graph in which each vertex has an even degree, there exists an Euler tour which traverses each edge of the component exactly once. Since the set of all edges in E_1 is a perfect matching of the bipartite graph G, as is the set of all edges in E_2, we know from Lemma 1 that the two partial permutations that correspond to the edges in E_1 and E_2 respectively, will eliminate crosstalk in the first stage and the last stage of the network. Of course, to get CFPPs, we need to eliminate crosstalk in all stages. As indicated in [7], a CFPP decomposition algorithm can be obtained by the repetition of the Euler Split procedure. A high-level description of the complete CFPP decomposition algorithm can be summarized as:

CFPP Decomposition Algorithm:

Initiate: i = 0 and take the permutation as the 0-level partial permutation.
Step 1: If $i = \lfloor (1/2)(\log_2 N+1) \rfloor$, exit.
Step 2: For each i-level partial permutation, do step 3 and step 4.
Step 3: Construct the bipartite graph $G = (V_1, V_2 ; E)$ for the i-level partial permutation. The vertex sets of G are defined as

$$V_1 = \left\{I_0, I_1, \cdots, I_{\frac{N}{2^{i+1}}-1}\right\},\ V_2 = \left\{O_0, O_1, \cdots, O_{\frac{N}{2^{i+1}}-1}\right\}$$

Here $I_j = \left\{u_{2^i \cdot j}, u_{2^i \cdot j+1}, \cdots, u_{2^i \cdot j + 2^i - 1}\right\}$ and $O_j = \left\{v_{2^i \cdot j}, v_{2^i \cdot j+1}, \cdots, v_{2^i \cdot j + 2^i - 1}\right\}$ for $0 \leq j \leq \frac{N}{2^{i+1}} - 1$, and u_0, u_1 ,…, $u_{N/2-1}$ are the N/2 inputs switches and v_0, v_1 ,…,$v_{N/2-1}$ are the N/2 outputs switches. The edge set E is defined as: for any one-pair mapping $\begin{pmatrix} x_i \\ y_i \end{pmatrix}$ in the i-level partial permutation, if the input switch corresponding to x_i belongs to I_{j_1} and the

output switch corresponding to y_i belongs to O_{j_2}, then there is an edge between vertex I_{j_1} and vertex O_{j_2} in *E*.
Step 4: Same as the Step 2 and Step 3 in the Euler Split Algorithm.
Step 5: Take all one-pair mappings corresponding to the edges in E_1, to form one (*i*+1)-level partial permutation corresponding to the *i*-level partial permutation; let the remaining one-pair mappings, corresponding to the edges in E_2, form another (*i*+1)-level partial permutation corresponding to the *i*-level partial permutation.
Step 6: $i \Leftarrow i+1$. Go to Step 1.

It is clear that after running Steps 2-5 for the *i*-level partial permutations of a permutation, the (*i*+1)-level partial permutations obtained will eliminate crosstalk in both the first (*i*+1) stages and the last (*i*+1) stages as guaranteed by Lemma 1. Thus, after running the decomposition algorithm for a permutation in a VSOB(*N*) network, the permutation will be decomposed into $2^{\lfloor (1/2)(\log_2 N+1)\rfloor}$ partial permutations. Since each of these partial permutations has $N/2^{\lfloor (1/2)(\log_2 N+1)\rfloor}$ elements and eliminates the crosstalk in all stages of the network, they actually are the CFPPs for the network. It is easy to see that the time to construct the bipartite graph is proportional to the number of pairs in the permutation, i.e., *O*(*N*), and the time to traverse all edges is *O*(*N*). So, Steps 2-5 take *O*(*N*) steps, and these steps will repeat $O(\log_2 N)$ times. Thus, the time complexity of the decomposition algorithm is $O(N\log N)$.

Example 2: The decomposition of the permutation in Example 1 into CFPPs.

Since *N*=16 and $(1/2)(\log_2 N)=2$ here, we need two levels of decomposition to decompose the permutation into $\sqrt{N}=4$ CFPPs.

First-level decomposition: Take the permutation in Example 1 as the 0-level partial permutation. The bipartite graph and edge traverses are shown in Fig.4, where

$$e_0=\begin{pmatrix}0\\8\end{pmatrix}, e_1=\begin{pmatrix}1\\2\end{pmatrix}, e_2=\begin{pmatrix}2\\0\end{pmatrix}, e_3=\begin{pmatrix}3\\12\end{pmatrix}, e_4=\begin{pmatrix}4\\4\end{pmatrix}, e_5=\begin{pmatrix}5\\13\end{pmatrix}, e_6=\begin{pmatrix}6\\3\end{pmatrix}, e_7=\begin{pmatrix}7\\11\end{pmatrix}, e_8=\begin{pmatrix}8\\9\end{pmatrix}, e_9=\begin{pmatrix}9\\1\end{pmatrix}, e_{10}=\begin{pmatrix}10\\6\end{pmatrix},$$
$$e_{11}=\begin{pmatrix}11\\7\end{pmatrix}, e_{12}=\begin{pmatrix}12\\5\end{pmatrix}, e_{13}=\begin{pmatrix}13\\10\end{pmatrix}, e_{14}=\begin{pmatrix}14\\15\end{pmatrix}, e_{15}=\begin{pmatrix}15\\14\end{pmatrix}.$$

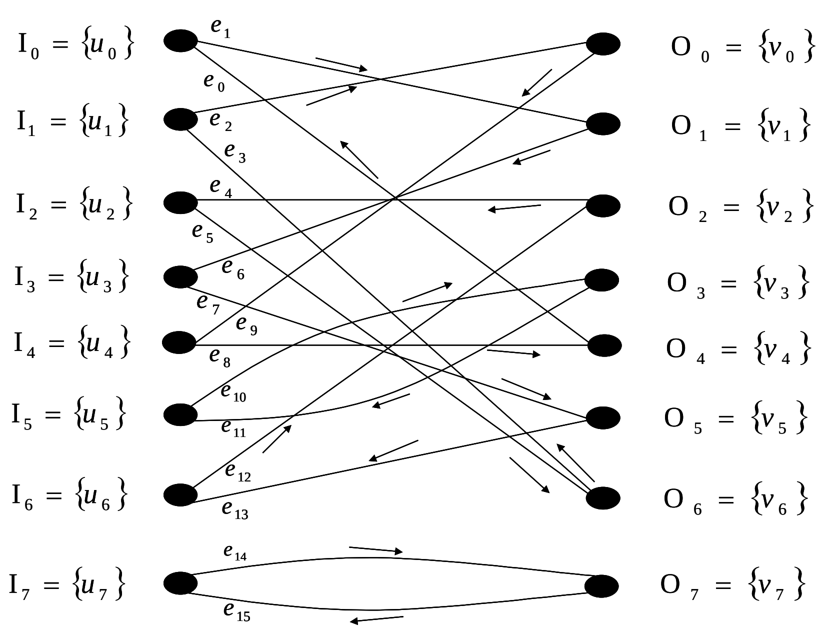


Fig.4 The bipartite graph and edge traverses of the First-level decomposition

Then the pairs e_1, e_2, e_5, e_7, e_8, e_{10}, e_{12} and e_{14} corresponding to the edges in E_1 form

$$\begin{pmatrix} 1 & 2 & 5 & 7 & 8 & 10 & 12 & 14 \\ 2 & 0 & 13 & 11 & 9 & 6 & 5 & 15 \end{pmatrix} \tag{11}$$

and the pairs e_0, e_3, e_4, e_6, e_9, e_{11}, e_{13} and e_{15} corresponding to the edges in E_2 form

$$\begin{pmatrix} 0 & 3 & 4 & 6 & 9 & 11 & 13 & 15 \\ 8 & 12 & 4 & 3 & 1 & 7 & 10 & 14 \end{pmatrix} \tag{12}$$

Second-level decomposition: For the first-level partial permutation (11), the bipartite graph and edge traverses are shown in Fig.5,

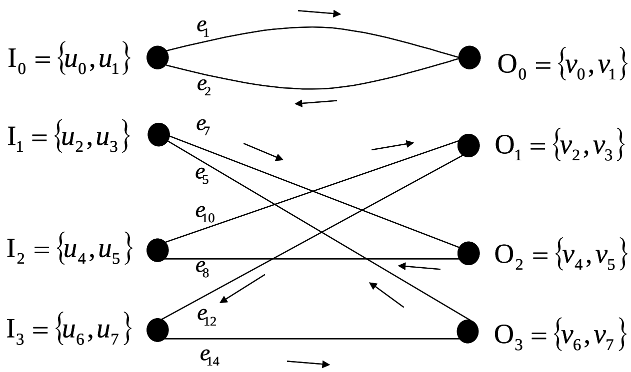


Fig.5 The first bipartite graph and edge traverses of the second-level decomposition

Then the pairs e_1, e_7, e_{10} and e_{14} corresponding to the edges in E_1 form

$$\begin{pmatrix} 1 & 7 & 10 & 14 \\ 2 & 11 & 6 & 15 \end{pmatrix} \tag{13}$$

and the pairs e_2, e_5, e_8 and e_{12} corresponding to the edges in E_2 form

$$\begin{pmatrix} 2 & 5 & 8 & 12 \\ 0 & 13 & 9 & 5 \end{pmatrix} \tag{14}$$

For the first-level partial permutation (12), the bipartite graph and edge traverses are shown in Fig.6,

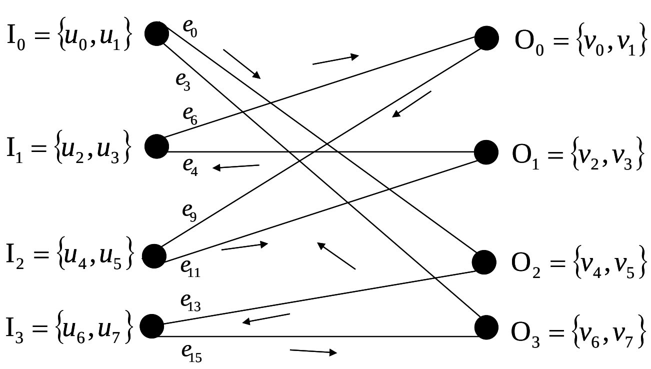


Fig.6 The second bipartite graph and edge traverses of the second-level decomposition

Then the pairs e_0, e_6, e_{11} and e_{15} corresponding to the edges in E_1 form

$$\begin{pmatrix} 0 & 6 & 11 & 15 \\ 8 & 3 & 7 & 14 \end{pmatrix} \tag{15}$$

and the pairs e_3, e_4, e_9 and e_{13} corresponding to the edges in E_2 form

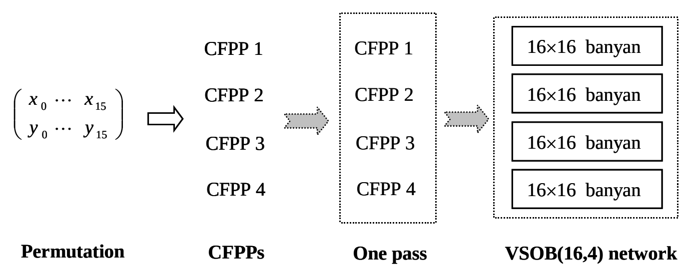
$$\begin{pmatrix} 3 & 4 & 9 & 13 \\ 12 & 4 & 1 & 10 \end{pmatrix} \quad (16)$$

which completes the whole decomposition. The four CFPPs are just the partial permutations (13),(14),(15) and (16) . By realizing each of the four CFPPs in a single plane of a VSOB(16) network, we can perform crosstalk-freely the permutation of Example 1 in a single pass based on parallel message transmission.

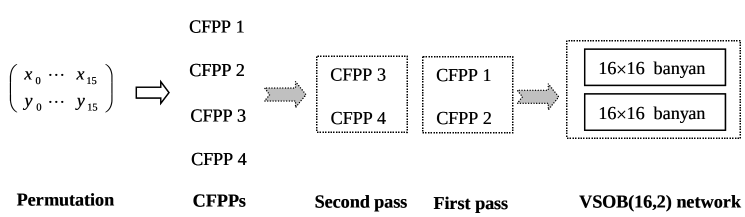
4. Number of planes vs. number of passes

Banyan networks have a simple switch setting ability (self-routing) and also a small number of SEs along a path between an input-output pair. These characteristics are important for DC-based optical switching networks, because loss and attenuation of an optical signal are proportional to the number of couplers that the optical signal passes through. The VSOB is an attractive structure for constructing optical switching networks without increasing the number of stages or sacrificing the self-routing property of banyan networks.

The result in Theorem 1 indicates that, to perform a crosstalk-free permutation of an N-element set $\{0,1,\ldots,N-1\}$, we can use serial message transmission that requires as many as $2^{\lfloor (1/2)(\log_2 N+1)\rfloor}$ passes in one $N \times N$ banyan network or use parallel message transmission that needs just one pass in a VSOB(N) network but requires as many as $2^{\lfloor (1/2)(\log_2 N+1)\rfloor}$ $N \times N$ banyan networks. Because of the good properties of a VSOB (it consists of multiple planes of identical banyan networks and each CFPP is crosstalk-free realizable in any of its planes), in general we can perform a crosstalk-free permutation in an $N \times N$ VSOB network in 2^i passes (i=0,1,…, $\lfloor (\log_2 N+1)/2 \rfloor$) if the network consists of $2^{\lfloor (1/2)(\log_2 N+1)\rfloor}/2^i$ planes. Thus, the structure of a VSOB network is flexible in that it enables a trade-off between the number of planes (hardware cost) and number of passes (transmission time) to be made based on a combination of serial and parallel message transmissions. Fig.7 illustrates the crosstalk-free permutation in two 16×16 VSOB networks with different numbers of passes and planes.



(a)



(b)

Fig.7 Illustration of crosstalk-free permutation in 16×16 VSOB networks. (a) Crosstalk-free permutation in VSOB(16,4) network in one pass. (b) Crosstalk-free permutation in VSOB(16,2) network in two passes.

For a large optical switching network, the tradeoff between the number of planes and the number of passes can be very significant because hardware cost can be reduced dramatically if a little transmission time is sacrificed. For example, for a 4096×4096 network, a VSOB(4096) network needs 64 planes to perform a crosstalk-free permutation in one pass, but this number of planes can be reduced to 32 if we allow two passes and 16 if we allow four passes to perform the same permutation. Thus, a tradeoff can be made between the number of planes and the number of passes, depending on the requirements of hardware cost and transmission time.

5. Conclusion

In this paper, we studied the crosstalk-free permutation capability of a vertically stacked optical banyan (VSOB) network applying packing strategy. We have shown that the nonblocking condition of a VSOB network under packing strategy is the same as the nonblocking condition of a rearrangeable VSOB network and that this nonblocking VSOB network is optimal in the sense that it consists of the minimum number of planes required by a nonblocking VSOB network. We proved that a permutation can be evenly decomposed into multiple crosstalk-free partial permutations (CFPPs), each of which is crosstalk-free realizable in one stacked plane of a VSOB network; and we also provided a complete algorithm for CFPP decomposition. We have shown that it is possible to trade the number of passes for the number of planes required to realize a crosstalk-free permutation in a VSOB network, and that this tradeoff can be very significant for a large optical switching network. It is expected that a VSOB network will have a high performance in fault tolerance because it consists of multiple copies of the same banyan network. We will examine this expectation in our future research.

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