

# Strictly Nonblocking $f$ -Cast Photonic Networks

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**Abstract**—The multicast capability and crosstalk issue need to be deliberately considered in the design of future high performance photonic switching networks. In this paper, we focus on the photonic switching networks built on the banyan-based architecture and directional coupler technology. We explore the capability of these networks to support general  $f$ -cast traffic, which covers the unicast traffic ( $f = 1$ ) and multicast traffic ( $f = N$ ) as special cases, and determine the conditions for these networks to be  $f$ -cast strictly nonblocking under various crosstalk constraints. In particular, we propose an optimization framework to determine the nonblocking condition of an  $f$ -cast photonic network when a general crosstalk constraint is imposed.

**Index Terms**—Banyan networks, crosstalk,  $f$ -cast, multicast, photonic switches, strictly nonblocking.

## I. INTRODUCTION

AS A RESULT of wavelength division multiplexing (WDM) technology, the number of wavelengths per fiber has been increased to hundreds or more with each wavelength operating at rates of 10 Gbit/s or higher [1]. Thus, the optical mesh network based on WDM technology becomes a promising backbone network of next generation Internet to meet the exponential growth in bandwidth demand from large numbers of users in multimedia applications, scientific computing, academic communities and the military. It is expected that the traffic carried on tens of fibers at each node in a WDM mesh network will soon approach several terabits per second. Switching such a huge amount of traffic electronically becomes very challenging, due to both the high cost of optical-electronic-optical conversion and the high costs related to heat dissipation and space consumption. Therefore, the adoption of all-optical photonic switching networks in WDM networks has been an active research area. Photonic switching networks not only have the potential to steer network traffic at the speed of hundreds of terabit per second or higher [2], but they also can be more cost-effective than their electronic counterparts, even for applications requiring lower throughput.

Combining the directional coupler (DC) technology [3], [4] and topologies of banyan class networks [5], [6] is an attractive approach to constructing photonic switching networks,

because a basic  $2 \times 2$  DC can simultaneously switch multiple wavelengths with high speed (on the order of nanosecond) and banyan networks have a small network depth as well as an absolute signal loss uniformity. However, with a banyan topology, only a unique path can be found from each network input to each network output, which degrades the network to a blocking one. A general approach to building banyan-based photonic switching networks (switches) is to jointly perform horizontal expansion (HE) and vertical stacking [VS] [7], [8], in which a regular banyan network is first horizontally expanded by adding some extra stages, and then multiple copies of the horizontally expanded banyan network are vertically stacked.

The study of banyan-based photonic switches has attracted extensive research activities, see, for example, [7]–[15]. Available results on the study of these networks can be roughly divided into two categories, the results about nonblocking conditions, such as [7]–[12], and the results about blocking behavior analysis, such as [13]–[15]. This paper focuses on the study of nonblocking conditions.

A nonblocking network can be either rearrangably nonblocking (RNB), wide-sense nonblocking (WNB) or strictly nonblocking (SNB). In a RNB network we can route any idle input to any idle output, but one or more existing connections may have to be rerouted. The design of RNB banyan-based photonic switches (with both VS and HE) has been addressed in [8], in which the strict crosstalk-free constraint was imposed. In a WNB network we can establish a connection between an idle input-output pair without disturbing the existing ones if a rule is followed during the connections setup. The paper [11] studied the WNB photonic switches (with VS only) under general crosstalk constraint, where a simple rule was proposed for connection setup. In a SNB network, we can always establish a connection between an idle input-output pair, regardless of how other connections are established. In paper [7], the authors explored the principles of constructing the general banyan-based SNB photonic switches under various crosstalk constraints. Our interest of this paper is on the general banyan-based SNB photonic switches.

The available study for nonblocking analysis of banyan-based photonic switches mainly focus on the one-to-one request (unicast), in which each input can request only one output. Due to the emerging applications of High Definition Television (HDTV), video-on-demand, video-conference, on-line gaming, etc., the connections from on one input to multiple outputs (multicast) or even to all outputs (broadcast) will be required. Recently, F.K.Hwang extended the study of banyan-based networks to the general  $f$ -cast case, in which an input can simultaneously request up to  $f$  distinct outputs [16]–[20]. The general  $f$ -cast covers the unicast case ( $f = 1$ ) and multicast case ( $f = N$ ) as special cases. It is notable that current research about supporting multicast (or more general

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$f$ -cast) in banyan-based switching network focuses mainly on the electronic networks [16], [20]–[22], in which only the link-blocking is involved in the analysis. For the banyan-based photonic switching networks with general crosstalk constraints, however, both link-blocking and crosstalk-blocking (caused by crosstalk constraints) can happen. It is the combination of link-blocking and crosstalk-blocking that makes the analysis of photonic switches different from their electronic counterparts. To the best of our knowledge no study is available about how to design multicast (or more generally  $f$ -cast) photonic switches when various crosstalk constraints are imposed. Thus this paper is committed to the study of banyan-based SNB photonic switches with both general  $f$ -cast requests and general crosstalk constraints. We will extend F.K.Hwang’s arguments for  $f$ -cast electronic networks with only link-blocking [16], [20] and Lea’s arguments for the photonic ones with only unicast traffic ( $f = 1$ ) [7] to study the general banyan-based  $f$ -cast photonic networks with both link-blocking and crosstalk-blocking. In particular we will develop a novel optimization framework to determine the conditions for these  $f$ -cast networks to be SNB when a general crosstalk constraint is imposed. Our study of this paper covers multicast ( $f = N$ ) and unicast ( $f = 1$ ) as our special cases.

The remainder of this paper is organized as follows. Section II introduces the preliminaries that will facilitate our further discussion. Sections III and IV present the  $f$ -cast SNB conditions for the banyan-based photonic switches without and with horizontal expansion, respectively. Section V provides the comparison among different switches, and finally Section VI concludes the paper.

## II. PRELIMINARIES

A typical  $N \times N$  banyan network consists of  $n = \log_2 N$  stages, each containing  $N/2 \times 2 \times 2$  switching elements and the link connections between adjacent stages are implemented by recursively applying the unshuffle interconnection pattern, as illustrated in Fig. 1. To construct a banyan-based switching network, a general approach is to first horizontally expand a  $N \times N$  banyan network by appending  $m$  ( $1 \leq m \leq \log_2 N - 1$ ) extra stages to the back of the network (as shown in Fig. 2 for  $N = 32$  and  $m = 2$ ), and then vertically stack  $K$  copies of the horizontally expanded network [7], [8], [23], as illustrated in Fig. 3. This class of networks covers many famous networks as its special cases, such as the Benes networks [24] and Cantor networks [25].

The banyan-based photonic switches are usually built on the directional coupler (DC) technology [3], [4], in which a basic switching element (SE) is implemented by a  $2 \times 2$  DC. A DC can simultaneously switch optical flows with the speed of some terabits per second and with multiple wavelengths, so it is one of the promising candidates to serve as the SE for future optical switching networks to support Optical Burst Switching and Optical Packet Switching. It is notable, however, that DC-based optical switching networks suffer from an intrinsic crosstalk problem [26], which happens when two optical signals pass through a SE at the same time. We call the SE that suffers from crosstalk as “crosstalk SE” (CSE). Thus, we can constrain the

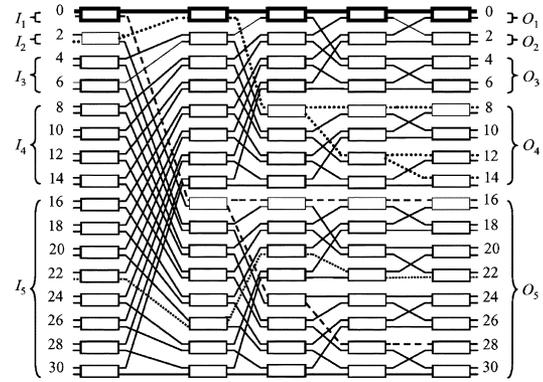


Fig. 1. A  $32 \times 32$  banyan network in which the tagged path between the input 0 and output 0 is highlighted.

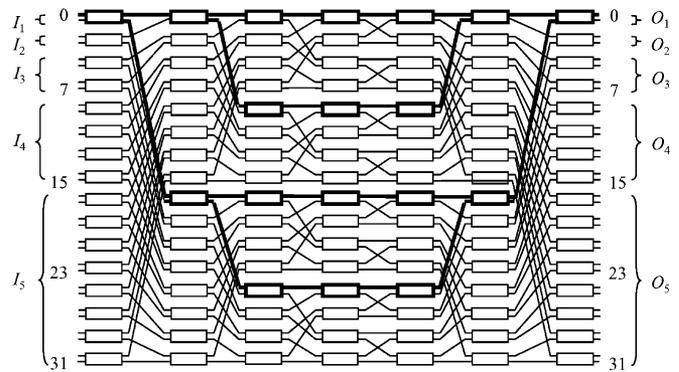


Fig. 2. A  $32 \times 32$  banyan network with two extra stages, in which the four tagged paths between the input 0 and output 0 are highlighted.

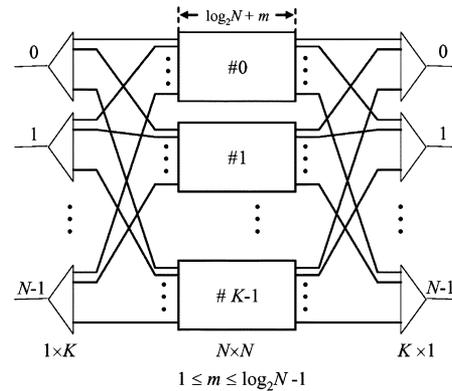


Fig. 3. General banyan-based switches constructed from the combination of horizontal expansion and vertical stacking schemes.

total amount of crosstalk of a connection by simply controlling the number of CSEs along the path of the connection<sup>1</sup> [7].

For the analysis of conventional electronic switches, we only need to address the link-blocking issue caused by the conflict when two signals try to go through a common link simultaneously. In a DC-based photonic switch, however, another kind of blocking is also relevant due to the new crosstalk constraint.

<sup>1</sup>We can also control the crosstalk within each SE by adopting a more complex SE architecture [12]. Here, we just focus on the simple DC-based SE architecture.

It can happen that all the links along the path of a new connection are free, but this connection will still be blocked because accepting this new connection may violate our crosstalk constraint (in terms of the total number of CSEs allowed along the path of either the new or an old connection.) We call this kind of blocking as crosstalk-blocking throughout this paper. The combination of link-blocking and crosstalk-blocking makes the analysis of a photonic switch different from its electronic counterpart with only link-blocking. Hereafter, we use the notation  $\log_2(N, m, K, c)$  to refer a general  $N \times N$  banyan-based switching network that consists  $m$  extra stage(s),  $K$  vertical copies (planes), and allows up to  $c$  CSEs along the path of any connection. We will determine the conditions for a  $\log_2(N, m, K, c)$  network to be SNB when the general  $f$ -cast requests are considered.

Due to the topological symmetry of a banyan network, all paths in it have the same property in terms of blocking. Based on the methodology established in [7], we can conduct the blocking analysis of a  $\log_2(N, m, K, c)$  network by focusing a tagged path and its associated input intersecting sets (IIS) and output intersecting sets (OIS). For a tagged path, all the SEs and links on the tagged path are called *tagged SEs* and *tagged links*, respectively. The stages of SEs and links are numbered from left (stage 1) to right (stage  $n$ ). For the tagged path between the input 0 and output 0 (please refer to Fig. 1), the IIS  $I_i = \{2^{i-1}, 2^{i-1} + 1, \dots, 2^i - 1\}$  is defined as the set of all inputs that intersect a tagged SE, for the first time, at stage  $i$ ; Symmetrically, the OIS  $O_i = \{2^{i-1}, 2^{i-1} + 1, \dots, 2^i - 1\}$  is the set of all outputs that intersect a tagged SE at stage  $n - i + 1, 1 \leq i \leq n$ . Fig. 1 illustrates the case of an  $f$ -cast  $32 \times 32$  banyan network with  $n = 5, f = 3$  and  $\lceil n - \log_2 f \rceil = 4$ , where the maximum fan-out of each input is limited to 3. Two multicast sessions  $\langle 1, \{16, 28\} \rangle, \langle 3, \{8, 12, 14\} \rangle$  and one unicast session  $\langle 23, 23 \rangle$  are shown by dashed and dotted lines.

### III. NETWORKS WITHOUT EXTRA STAGES

For a network without extra stages, each plane has one unique path for each input-output pair and the connection between this input-output pair can be established through this plane if its corresponding path is free.

Since the requests of a request session<sup>2</sup> may be routed independently through different planes. Therefore, we just need to focus on only one of these requests in our analysis and we regard the path of the selected request as the tagged path. Here, we focus on the tagged path between input 0 and output 0. Notice that the requests from the same session cannot block each other, since they can share SEs and links. Thus, we only need to consider the requests from sessions other than that of the tagged path in our blocking analysis.

#### A. Strictly Nonblocking $f$ -Cast $\log_2(2^n, 0, K, 0)$ Networks

For a  $\log_2(2^n, 0, K, 0)$  network with the crosstalk-free constraint ( $c = 0$ ), we allow only one signal to pass through a SE at a time and crosstalk blocking (or node blocking) will happen

<sup>2</sup>For a  $f$ -cast network, we define a *request session* of the network as the set of requests that originate from a common input and are destined for at most  $f$  distinct outputs.

when two signals need to pass through a common SE simultaneously. The conditions for a  $\log_2(2^n, 0, K, 0)$  network to be  $f$ -cast SNB is summarized in the following theorem.

*Theorem 1:* A  $\log_2(2^n, 0, K, 0)$  network is  $f$ -cast SNB for  $1 \leq f < 2^n$  if and only if (iff)

$$K \geq \begin{cases} \left( \sqrt{2^{\lceil n - \log_2 f \rceil}} - 1 \right) \cdot f + \sqrt{2^{\lceil n + \log_2 f \rceil}}, & \text{if } \lceil n - \log_2 f \rceil \text{ is even,} \\ \left( \sqrt{2^{\lceil n - \log_2 f \rceil - 1}} - 1 \right) \cdot f + \sqrt{2^{\lceil n + \log_2 f \rceil + 1}}, & \text{if } \lceil n - \log_2 f \rceil \text{ is odd.} \end{cases}$$

*Proof:* Under the crosstalk-free constraint ( $c = 0$ ), we will focus on the tagged SEs and consider only the crosstalk blocking (node blocking) in our analysis. For a given  $f (1 \leq f < 2^n)$  and when  $\lceil n - \log_2 f \rceil$  is even (please refer to Fig. 1), we can determine a unique integer  $j$  such that<sup>3</sup>

$$2^{n-2j-2} \leq f < 2^{n-2j-1} \text{ or} \\ |O_{n-(j+1)}| \leq |I_{j+1}| \cdot f < |O_{n-j}|. \quad (1)$$

We can prove easily that the parameter  $j$  in (1) is given by the following formula: (For the sake of presentation, full proofs of some results in this paper are presented in [27].)

$$j = \lceil n - \log_2 f \rceil / 2 - 1. \quad (2)$$

Since the upper bound in (1) indicates that  $f < 2^{2i-1}$  for  $1 \leq i \leq j$ , thus, we have

$$|I_i| \cdot f < 2^{i-1} \cdot 2^{n-2i-1} < 2^{n-i} = |O_{n-i+1}|, \\ \text{for } 1 \leq i \leq j. \quad (3)$$

Expressions (1) and (3) imply that from stage 1 to stage  $j+1$ , the maximum number of conflicts with the tagged path is just  $\sum_{i=1}^{j+1} |I_i| \cdot f$ . The lower bound in (1) indicates that

$$|I_{j+2}| \cdot f \geq 2^{j+2-1} \cdot 2^{n-2j-2} > \sum_{k=1}^{n-(j+2)+1} |O_k|.$$

Thus, from stage  $j+2$  to stage  $n$  the maximum number of conflicts with the tagged path is determined by the remaining outputs  $\sum_{k=1}^{n-(j+2)+1} |O_k| = \sum_{k=1}^{n-(j+1)} |O_k|$ . Therefore, the total number of conflicts (blocked planes) of the tagged path is

$$\sum_{i=1}^{j+1} |I_i| \cdot f + \sum_{k=1}^{n-(j+1)} |O_k| = (2^{j+1} - 1) \cdot f + 2^{n-j-1} - 1. \quad (4)$$

By applying formula (2) to the above equation and adding one extra plane for the tagged connection, we can see that when  $\lceil n - \log_2 f \rceil$  is even, the total number of planes required for a SNB  $\log_2(2^n, 0, K, 0)$  network is  $(\sqrt{2^{\lceil n - \log_2 f \rceil}} - 1) \cdot f + \sqrt{2^{\lceil n + \log_2 f \rceil}}$ .

For the case when  $\lceil n - \log_2 f \rceil$  is odd, we can also determine a unique integer  $j$  such that

$$2^{n-2j-1} \leq f < 2^{n-2j} \text{ or} \\ |O_{n-j}| \leq |I_{j+1}| \cdot f < |O_{n-j+1}|. \quad (5)$$

<sup>3</sup>The condition (2) actually corresponds to a partition of interval  $[1, 2^n)$ . For a given  $f \in [1, 2^n)$ , we can determine a unique interval  $[2^{n-2j-2}, 2^{n-2j-1})$  with index  $j$  such that  $f$  falls within it.

We can see that the unique  $j$  in (5) is determined as

$$j = (\lceil n - \log_2 f \rceil - 1)/2. \quad (6)$$

Following a similar discussion as that of the case when  $\lceil n - \log_2 f \rceil$  is even, we can prove that the total number of blocked planes of the tagged path is now given by

$$\sum_{i=1}^j |I_i| \cdot f + \sum_{k=1}^{n-j} |O_k| = (2^j - 1) \cdot f + 2^{n-j} - 1 \quad (7)$$

Thus, the total number of planes required for a SNB  $\log_2(2^n, 0, K, 0)$  network is  $(\sqrt{2^{\lceil n - \log_2 f \rceil - 1}} - 1) \cdot f + \sqrt{2^{\lceil n + \log_2 f \rceil + 1}}$  for the case when  $\lceil n - \log_2 f \rceil$  is odd.

It is notable that the conditions in (4) and (7) are also necessary, because these maximum numbers of blocked planes can be achieved by simply creating a worst-case request pattern according to the case assumed in the proof. QED.

When we set  $f = 1$  (unicast) or  $f = 2^n - 1$  (multicast) in Theorem 1, we can achieve the nonblocking conditions for a unicast  $\log_2(2^n, 0, K, 0)$  network [7] or for a multicast  $\log_2(2^n, 0, K, 0)$  network, respectively.

*Corollary 3.1:* A  $\log_2(2^n, 0, K, 0)$  network is unicast SNB iff

$$K \geq \begin{cases} 2\sqrt{2^n} - 1, & \text{if } n \text{ is even,} \\ (3/2)\sqrt{2^{n+1}} - 1, & \text{if } n \text{ is odd.} \end{cases}$$

*Corollary 3.2:* A  $\log_2(2^n, 0, K, 0)$  network is multicast SNB iff

$$K > 2^n.$$

### B. Strictly Nonblocking $f$ -Cast $\log_2(2^n, 0, K, n)$ Networks

For a  $\log_2(2^n, 0, K, n)$  network without crosstalk constraint, only link-blocking can happen and the condition for the network to be SNB has been addressed by F. K. Hwang [16]. Here, we present the condition in a similar format as that of Theorem 1 depending whether  $\lceil n - \log_2 f \rceil$  is even or odd. This format will be adopted for presenting the nonblocking conditions of general  $\log_2(2^n, 0, K, c)$  networks in Section C.

*Theorem 2:* A  $\log_2(2^n, 0, K, n)$  network is  $f$ -cast SNB for  $1 \leq f < 2^n$  iff

$$K \geq \begin{cases} \left( (1/2)\sqrt{2^{\lceil n - \log_2 f \rceil}} - 1 \right) \cdot f + \sqrt{2^{\lceil n + \log_2 f \rceil}}, & \text{if } \lceil n - \log_2 f \rceil \text{ is even,} \\ \left( \sqrt{2^{\lceil n - \log_2 f \rceil - 1}} - 1 \right) \cdot f + \sqrt{2^{\lceil n + \log_2 f \rceil - 1}}, & \text{if } \lceil n - \log_2 f \rceil \text{ is odd.} \end{cases}$$

*Proof:* For a  $\log_2(2^n, 0, K, n)$  network that has no crosstalk constraint, we only need to consider the link-blocking in the analysis. For a given  $f (1 \leq f < 2^n)$ , we can determine a unique integer  $j$  by (2) or (6) depending on whether  $\lceil n - \log_2 f \rceil$  is even or odd. Following a similar proof as that of Theorem 1, we can see that the nonblocking condition for a  $\log_2(2^n, 0, K, n)$  network is

$$\begin{aligned} K &\geq \left( \sum_{i=1}^j |I_i| \cdot f + \sum_{k=1}^{n-j-1} |O_k| \right) + 1 \\ &= (2^j - 1) \cdot f + 2^{n-j-1} \end{aligned}$$

$$= \begin{cases} \left( (1/2)\sqrt{2^{\lceil n - \log_2 f \rceil}} - 1 \right) \cdot f + \sqrt{2^{\lceil n + \log_2 f \rceil}}, & \text{if } \lceil n - \log_2 f \rceil \text{ is even,} \\ \left( \sqrt{2^{\lceil n - \log_2 f \rceil - 1}} - 1 \right) \cdot f + \sqrt{2^{\lceil n + \log_2 f \rceil - 1}}, & \text{if } \lceil n - \log_2 f \rceil \text{ is odd.} \end{cases} \quad (8)$$

Again, if we set  $f = 1$  or  $f = 2^n - 1$  in Theorem 2, we can achieve the nonblocking conditions for a unicast  $\log_2(2^n, 0, K, n)$  network or for a multicast  $\log_2(2^n, 0, K, n)$  network, respectively.

*Corollary 3.3:* A  $\log_2(2^n, 0, K, n)$  network is unicast SNB iff

$$K \geq \begin{cases} (3/2)\sqrt{2^n} - 1, & \text{if } n \text{ is even,} \\ \sqrt{2^{n+1}} - 1, & \text{if } n \text{ is odd.} \end{cases}$$

*Corollary 3.4:* A  $\log_2(2^n, 0, K, n)$  network is multicast SNB iff

$$K \geq 2^{n-1}.$$

### C. Strictly Nonblocking $f$ -Cast $\log_2(2^n, 0, K, c)$ Networks

In a  $\log_2(2^n, 0, K, c)$  network with a general crosstalk constraint  $c (0 < c < n)$ , both the crosstalk-blocking and link-blocking can happen, and it is the combination of these two kinds of blocking that makes the analysis of such a network much complicated. Here, we propose an optimization framework to determine the nonblocking condition of an  $f$ -cast  $\log_2(2^n, 0, K, c)$  network.

*Theorem 3:* A  $\log_2(2^n, 0, K, c)$  network is  $f$ -cast SNB for  $1 \leq f < 2^n$  if the following conditions hold:

When  $\lceil n - \log_2 f \rceil$  is even:

$$K \geq \begin{cases} m_{\text{even}} + \left\lfloor \frac{m_1 + 2m_2}{c+2} \right\rfloor, & \text{if } 0 < c \leq \lfloor 2^{\lceil 1 - \log_2 f \rceil} \cdot f \rfloor - 1 \\ m_{\text{even}} + \left\lfloor \frac{m_1 + m_2 + \lfloor 2m_1 / (c+1) \rfloor}{c+2} \right\rfloor, & \text{if } \lfloor 2^{\lceil 1 - \log_2 f \rceil} \cdot f \rfloor - 1 < c < n. \end{cases} \quad (9)$$

where

$$\begin{aligned} m_{\text{even}} &= \left( (1/2)\sqrt{2^{\lceil n - \log_2 f \rceil}} - 1 \right) \cdot f + \sqrt{2^{\lceil n + \log_2 f \rceil}}, \\ m_1 &= (1/2)\sqrt{2^{\lceil n - \log_2 f \rceil}} \cdot f, \\ m_2 &= (1/2)\sqrt{2^{\lceil n + \log_2 f \rceil}}. \end{aligned} \quad (10)$$

When  $\lceil n - \log_2 f \rceil$  is odd, the condition becomes

$$K \geq \begin{cases} m_{\text{odd}} + \left\lfloor \frac{m_3 + 2m_4}{c+2} \right\rfloor, & \text{if } 0 < c \leq \lfloor 2^{\lceil 2 - \log_2 f \rceil} \cdot f \rfloor - 1 \\ m_{\text{odd}} + \left\lfloor \frac{m_3 + m_4 + \lfloor 2m_3 / (c+1) \rfloor}{c+2} \right\rfloor, & \text{if } \lfloor 2^{\lceil 2 - \log_2 f \rceil} \cdot f \rfloor - 1 < c < n \end{cases} \quad (11)$$

where

$$\begin{aligned} m_{\text{odd}} &= \left( \sqrt{2^{\lceil n - \log_2 f \rceil - 1}} - 1 \right) \cdot f + \sqrt{2^{\lceil n + \log_2 f \rceil - 1}}, \\ m_3 &= \sqrt{2^{\lceil n - \log_2 f \rceil - 1}} \cdot f, \\ m_4 &= (1/2)\sqrt{2^{\lceil n + \log_2 f \rceil - 1}}. \end{aligned} \quad (12)$$

*Proof:* For a general  $f$ -cast  $\log_2(2^n, 0, K, c)$  network that allows up to  $c(0 < c < n)$  CSEs along any connection, its non-blocking condition will be upper-bounded and lower-bounded by the results in Theorem 1 and in Theorem 2, respectively.

1) When  $\lceil n - \log_2 f \rceil$  is Even: For the case that  $\lceil n - \log_2 f \rceil$  is even (please refer to Fig. 4), we know from the Proof of Theorem 1 that we can determine a unique integer  $j$  by  $j = \lceil n - \log_2 f \rceil / 2 - 1$ , which implies that  $2^{n-2j-2} \leq f < 2^{n-2j-1}$  or  $|O_{n-(j+1)}| \leq |I_{j+1}|f < |O_{n-j}|$ . By comparing the conditions (4) for  $c = 0$  and (8) for  $c = n$  we can see that regardless of the blocking type (link-blocking or crosstalk-blocking) and parameter  $c$ , the connections from  $\bigcup_{i=1}^j I_i$  and the connections to  $\bigcup_{k=1}^{n-(j+2)} O_k$  can always block  $\sum_{i=1}^j |I_i| \cdot f + \sum_{k=1}^{n-(j+2)} |O_k|$  planes, and the difference between the two conditions is determined by the connections from  $I_{j+1}$  and connections to  $O_{n-(j+1)}$ . When  $c = 0$ , the connections from  $I_{j+1}$  to  $O_{n-j}$  can block  $\min(|I_{j+1}|f, |O_{n-j}|)$  planes (due to crosstalk-blocking), and similarly the connections from  $I_{j+2}$  to  $O_{n-(j+1)}$  can block  $\min(|I_{j+2}|f, |O_{n-(j+1)}|)$  planes. Since we now have  $|I_{j+1}|f < |O_{n-j}|$  and  $|I_{j+2}|f \geq |O_{n-(j+1)}|$  when  $\lceil n - \log_2 f \rceil$  is even, so for the crosstalk-free case ( $c = 0$ ) the total number of planes required for nonblocking is at least<sup>4</sup>

$$\begin{aligned} & \sum_{i=1}^j |I_i| \cdot f + \sum_{k=1}^{n-(j+2)} |O_k| + \min(|I_{j+1}|f, |O_{n-j}|) \\ & + \min(|I_{j+2}|f, |O_{n-(j+1)}|) + 1 \\ & = \sum_{i=1}^{j+1} |I_i| \cdot f + \sum_{k=1}^{n-(j+1)} |O_k| + 1. \end{aligned} \quad (13)$$

When  $c = n$  (no crosstalk constraint), only link-blocking can occur and the connections from  $I_{j+1}$  must be destined for  $O_{n-(j+1)}$  to create link-blocking for the tagged path. Thus, the total number of planes required for nonblocking will be given by the following equation (14) due to the fact that  $|I_{j+1}|f \geq |O_{n-(j+1)}|$ :

$$\begin{aligned} & \sum_{i=1}^j |I_i| \cdot f + \sum_{k=1}^{n-(j+2)} |O_k| \\ & + \min(|I_{j+1}| \cdot f, |O_{n-(j+1)}|) + 1 \\ & = \sum_{i=1}^j |I_i| \cdot f + \sum_{k=1}^{n-(j+1)} |O_k| + 1. \end{aligned} \quad (14)$$

For the general case that  $0 < c < n$ , however, both link-blocking and crosstalk-blocking<sup>5</sup> need to be considered in the blocking analysis. We shall determine the maximum number of planes that the connections from  $I_{j+1}$  and the connections to  $O_{n-(j+1)}$  can jointly block based on the combination of link-

<sup>4</sup>In addition to the blocked planes, we need one more plane for the new connection to guarantee nonblocking.

<sup>5</sup>We will consider only the case that adding the tagged connection will cause more than  $c$  CSEs along another connection intersecting the tagged path, since we have already assumed that in the worst case each connection from  $\bigcup_{i=1}^j I_i$  and the each connection to  $\bigcup_{k=1}^{n-(j+2)} O_k$  will block a distinct plane.

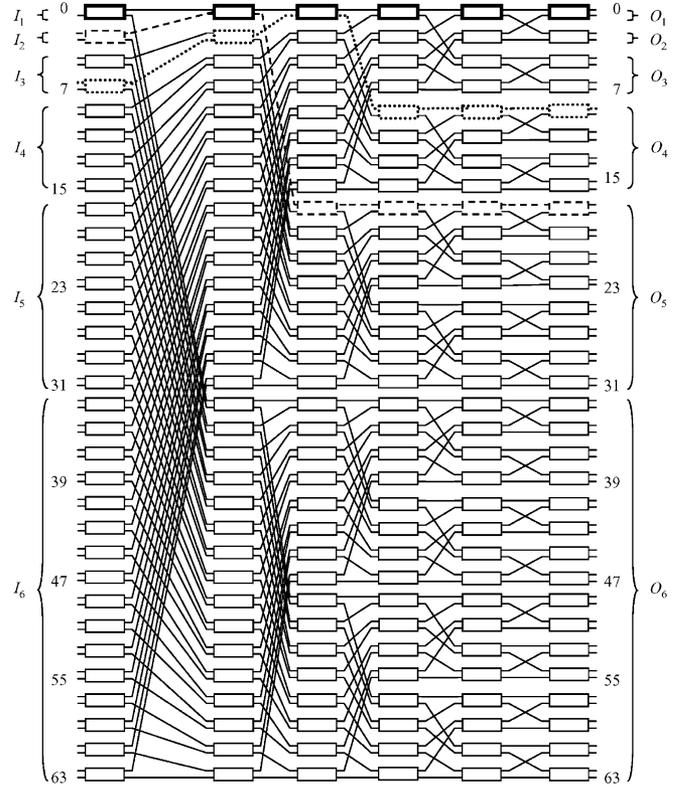


Fig. 4. Illustration of an  $f$ -cast  $64 \times 64$  banyan network with  $n = 6$ ,  $f = 6$ ,  $\lceil n - \log_2 f \rceil = 4$  and  $j = \lceil n - \log_2 f \rceil / 2 - 1 = 1$ . The dashed connection is from  $I_{j+1}(I_2)$  to  $O_{n-j}(O_5)$  while the dotted connection is from  $I_{j+2}(I_3)$  to  $O_{n-(j+1)}(O_4)$ , and we have  $|O_{n-(j+1)}| \leq |I_{j+1}| \cdot f < |O_{n-j}|$ .

blocking and crosstalk-blocking. Let  $x_1$  denotes the connections from  $I_{j+1}$  to  $O_{n-j}$  (such as the dashed connection in Fig. 4) and  $x_2$  denotes the connections from  $I_{j+2}$  to  $O_{n-(j+1)}$  (such as the dotted connection in Fig. 4) that incur the crosstalk-blocking to the tagged path, and let  $l$  denotes the connections from  $I_{j+1}$  to  $O_{n-(j+1)}$  that cause link-blocking to the tagged path. Then for a given  $c$ , an upper bound  $Z_{\text{even}}(c)$  on the maximum number of planes that the connections from  $I_{j+1}$  and the connections to  $O_{n-(j+1)}$  can jointly block is determined by

$$Z_{\text{even}}(c) = \max_l (l + x_1 + x_2) \quad (15)$$

$$\text{Subject to : } 0 \leq l \leq |O_{n-(j+1)}| \quad (16)$$

$$0 \leq x_1 \leq \min \left( |I_{j+1}| \cdot f - l, \left\lfloor \frac{|I_{j+1}| \cdot f - l + |O_{n-j}|}{c + 2} \right\rfloor \right) \quad (17)$$

$$0 \leq x_2 \leq \min \left( |O_{n-(j+1)}| - l, \left\lfloor \frac{|I_{j+2}| \cdot f + |O_{n-(j+1)}| - l}{c + 2} \right\rfloor \right). \quad (18)$$

The above three constraints are due to the facts that  $|O_{n-(j+1)}| \leq |I_{j+1}|f < |O_{n-j}|$  for the case  $\lceil n - \log_2 f \rceil$  is even and we need  $c + 2$  connections to cause one crosstalk-blocking<sup>6</sup>. Then a  $f$ -cast  $\log_2(2^n, 0, K, c)$  net-

<sup>6</sup>For a connection from  $I_{j+1}$  to  $O_{n-j}$  or from  $I_{j+2}$  to  $O_{n-(j+1)}$  to cause a crosstalk-blocking, two elements from both sets are needed to establish the connection and  $c$  other elements from the two sets are required to create  $c$  CSEs.

work with a general  $c(0 < c < n)$  and even  $\lceil n - \log_2 f \rceil$  is SNB if

$$K \geq \sum_{i=1}^j |I_i| \cdot f + \sum_{k=1}^{n-(j+2)} |O_k| + Z_{\text{even}}(c) + 1. \quad (19)$$

It is interesting to notice that the  $Z_{\text{even}}(c)$  in (15) satisfies the following constraints:

$$|O_{n-(j+1)}| \leq Z_{\text{even}}(c) \leq |O_{n-(j+1)}| + |I_{j+1}| \cdot f.$$

Thus, the condition  $\sum_{i=1}^j |I_i| \cdot f + \sum_{k=1}^{n-(j+2)} |O_k| + Z_{\text{even}}(c) + 1$  in (19) for general  $c$  is just upper-bounded by the condition (13) for  $c = 0$  and lower-bounded by the condition (14) for  $c = n$ . In the following, we shall derive the formula for  $Z_{\text{even}}(c)$  to get the nonblocking condition of the general  $\log_2(2^n, 0, K, c)$  networks.

To find the value of  $Z_{\text{even}}(c)$  in (15), we first need to simplify the constraints (17) and (18). Notice that

$$\begin{aligned} |I_{j+1}| \cdot f - l &\leq \left\lfloor \frac{|I_{j+1}| \cdot f - l + |O_{n-j}|}{c+2} \right\rfloor \Leftrightarrow l \\ &\geq |I_{j+1}| \cdot f - \left\lfloor \frac{|O_{n-j}|}{c+1} \right\rfloor \end{aligned} \quad (20)$$

or equivalently

$$\begin{aligned} |I_{j+1}| \cdot f - l &> \left\lfloor \frac{|I_{j+1}| \cdot f - l + |O_{n-j}|}{c+2} \right\rfloor \Leftrightarrow l \\ &< |I_{j+1}| \cdot f - \left\lfloor \frac{|O_{n-j}|}{c+1} \right\rfloor \end{aligned}$$

And

$$\begin{aligned} |O_{n-(j+1)}| - l &\leq \left\lfloor \frac{|I_{j+2}| \cdot f + |O_{n-(j+1)}| - l}{c+2} \right\rfloor \Leftrightarrow l \\ &\geq |O_{n-(j+1)}| - \left\lfloor \frac{|I_{j+2}| \cdot f}{c+1} \right\rfloor \end{aligned} \quad (21)$$

or equivalently

$$\begin{aligned} |O_{n-(j+1)}| - l &> \left\lfloor \frac{|I_{j+2}| \cdot f + |O_{n-(j+1)}| - l}{c+2} \right\rfloor \Leftrightarrow l \\ &< |O_{n-(j+1)}| - \left\lfloor \frac{|I_{j+2}| \cdot f}{c+1} \right\rfloor. \end{aligned}$$

Since we now have  $|O_{n-(j+1)}| \leq |I_{j+1}| \cdot f < |O_{n-j}|$ , then the term  $|I_{j+1}| \cdot f - \lfloor |O_{n-j}|/(c+1) \rfloor$  in (20) may be smaller or larger than  $|O_{n-(j+1)}|$  and the term  $|O_{n-(j+1)}| - \lfloor |I_{j+2}| \cdot f/(c+1) \rfloor$  in (21) may be larger or smaller than 0. Notice that the constraint (16) requires  $0 \leq l \leq |O_{n-(j+1)}|$ , so we separately consider the following, all the four possible cases about terms  $|O_{n-(j+1)}| - \lfloor |I_{j+2}| \cdot f/(c+1) \rfloor$  and  $|I_{j+1}| \cdot f - \lfloor |O_{n-j}|/(c+1) \rfloor$  to determine the  $Z_{\text{even}}(c)$ .

**Case 1:**  $|O_{n-(j+1)}| - \lfloor |I_{j+2}| \cdot f/(c+1) \rfloor > 0$  and  $|I_{j+1}| \cdot f - \lfloor |O_{n-j}|/(c+1) \rfloor \leq |O_{n-(j+1)}|$ .

For the network illustrated in Fig. 4, one configuration corresponding to this case is when  $c = 3$  and  $f = 6$ , then  $j = \lceil n - \log_2 f \rceil / 2 - 1 = 1$ . Thus, we have  $|O_{n-(j+1)}| - \lfloor |I_{j+2}| \cdot f/(c+1) \rfloor = |O_4| - \lfloor |I_3| \cdot 6/(3+1) \rfloor = 8 - 6 > 0$  and  $|I_{j+1}| \cdot f - \lfloor |O_{n-j}|/(c+1) \rfloor = |I_2| \cdot 6 - \lfloor |O_5|/(3+1) \rfloor = 8 \leq$

$|O_4| = |O_{n-(j+1)}|$ . Under *Case 1*, both inequalities (20) and (21) can hold depending on the value of  $l$ . Since we always have  $|I_{j+1}| \cdot f - \lfloor |O_{n-j}|/(c+1) \rfloor \geq |O_{n-(j+1)}| - \lfloor |I_{j+2}| \cdot f/(c+1) \rfloor$ , so we can get the value of  $Z_{\text{even}}(c)$  for this case by separating the whole range  $0 \leq l \leq |O_{n-(j+1)}|$  for  $l$  into the following three subranges and evaluating the term  $\max_l(l + x_1 + x_2)$  for each range.

• If  $|O_{n-(j+1)}| \geq l \geq |I_{j+1}| \cdot f - \lfloor |O_{n-j}|/(c+1) \rfloor$  (Range 1):

For the above configuration,  $|O_{n-(j+1)}| = |I_{j+1}| \cdot f - \lfloor |O_{n-j}|/(c+1) \rfloor = 8$ , so this subcase just corresponds to the range  $8 \geq l \geq 8$  (i.e.,  $l = 8$ ). Notice that when  $l$  falls within the subrange 1 that  $|O_{n-(j+1)}| \geq l \geq |I_{j+1}| \cdot f - \lfloor |O_{n-j}|/(c+1) \rfloor$ , both inequalities (20) and (21) hold. Thus, the value of  $\max_l(l + x_1 + x_2)$  in this subrange (denoted as  $Z_1$  here) is given by

$$\begin{aligned} Z_1 &= \max_{l \in \text{Range1}} \{l + (|I_{j+1}| \cdot f - l) \\ &\quad + (|O_{n-(j+1)}| - l)\} \\ &= |I_{j+1}| \cdot f + |O_{n-(j+1)}| \\ &\quad - (|I_{j+1}| \cdot f - \lfloor |O_{n-j}|/(c+1) \rfloor) \\ &= |O_{n-(j+1)}| + \lfloor |O_{n-j}|/(c+1) \rfloor. \end{aligned} \quad (22)$$

• If  $|I_{j+1}| \cdot f - \lfloor |O_{n-j}|/(c+1) \rfloor > l \geq |O_{n-(j+1)}| - \lfloor |I_{j+2}| \cdot f/(c+1) \rfloor$  (Range 2):

Again, for the above configuration we have  $|O_{n-(j+1)}| - \lfloor |I_{j+2}| \cdot f/(c+1) \rfloor = 2$ , so this subcase corresponds to the range  $8 > l \geq 2$ . For this subrange 2 that  $|I_{j+1}| \cdot f - \lfloor |O_{n-j}|/(c+1) \rfloor > l \geq |O_{n-(j+1)}| - \lfloor |I_{j+2}| \cdot f/(c+1) \rfloor$ , inequality (21) holds but inequality (20) does not. Thus, the value of  $\max_l(l + x_1 + x_2)$  in this range (denoted as  $Z_2$  here) is evaluated as

$$\begin{aligned} Z_2 &= \max_{l \in \text{Range2}} \left\{ l + \frac{\lfloor |I_{j+1}| \cdot f - l + |O_{n-j}| \rfloor}{c+2} \right. \\ &\quad \left. + (|O_{n-(j+1)}| - l) \right\} \\ &= |O_{n-(j+1)}| \\ &\quad + \left\lfloor \frac{|I_{j+1}| \cdot f - (|O_{n-(j+1)}| - \lfloor |I_{j+2}| \cdot f/(c+1) \rfloor) + |O_{n-j}|}{c+2} \right\rfloor \\ &= |O_{n-(j+1)}| \\ &\quad + \left\lfloor \frac{|I_{j+1}| \cdot f + |O_{n-(j+1)}| + \lfloor |I_{j+2}| \cdot f/(c+1) \rfloor}{c+2} \right\rfloor. \end{aligned} \quad (23)$$

Notice that  $|I_{j+1}| \cdot f \geq |O_{n-(j+1)}|$  and  $|I_{j+2}| \cdot f \geq |O_{n-j}|$  when  $\lceil n - \log_2 f \rceil$  is even, so we have

$$\begin{aligned} Z_2 &\geq |O_{n-(j+1)}| \\ &\quad + \left\lfloor \frac{|O_{n-(j+1)}| + |O_{n-(j+1)}| + \lfloor |O_{n-j}|/(c+1) \rfloor}{c+2} \right\rfloor \\ &\geq |O_{n-(j+1)}| \\ &\quad + \left\lfloor \frac{(c+1) \cdot \lfloor |O_{n-j}|/(c+1) \rfloor + \lfloor |O_{n-j}|/(c+1) \rfloor}{c+2} \right\rfloor \\ &= |O_{n-(j+1)}| + \lfloor |O_{n-j}|/(c+1) \rfloor = Z_1. \end{aligned} \quad (24)$$

• If  $|O_{n-(j+1)}| - \lfloor |I_{j+2}| \cdot f/(c+1) \rfloor > l \geq 0$  (Range 3):

Finally, for the above configuration, this subcase corresponds to the range  $2 > l \geq 0$ . When  $l$  falls within this range 3, neither inequality (20) nor inequality (21) holds. Therefore, the value of  $\max_l(l + x_1 + x_2)$  in this subrange (denoted as  $Z_3$  here) is determined as

$$Z_3 = \max_{l \in \text{Range3}} \left\{ l + \left\lfloor \frac{|I_{j+1}| \cdot f - l + |O_{n-j}|}{c+2} \right\rfloor + \left\lfloor \frac{|I_{j+2}| \cdot f + |O_{n-(j+1)}| - l}{c+2} \right\rfloor \right\}.$$

Since for any  $a > 0$  and  $b > 0$ , we know that

$$\lfloor a \rfloor + \lfloor b \rfloor = \begin{cases} \lfloor a + b \rfloor, & \text{if } \lfloor a \rfloor + \lfloor b \rfloor + \lfloor a + b \rfloor \text{ is even,} \\ \lfloor a + b \rfloor - 1, & \text{if } \lfloor a \rfloor + \lfloor b \rfloor + \lfloor a + b \rfloor \text{ is odd.} \end{cases}$$

Thus, the maximum possible value of  $Z_3$  is given by the equation shown at the bottom of the page. To show the relationship between  $Z_2$  and  $Z_3$ , we have

$$\begin{aligned} & \left( |I_{j+1}| \cdot f + |O_{n-(j+1)}| + \left\lfloor \frac{|I_{j+2}| \cdot f}{c+1} \right\rfloor \right) \\ & - (|I_{j+1}| \cdot f + |I_{j+2}| \cdot f + |O_{n-(j+1)}| \\ & - c \cdot \left\lfloor \frac{|I_{j+2}| \cdot f}{c+1} \right\rfloor - c) \\ & = (c+1) \cdot \left\lfloor \frac{|I_{j+2}| \cdot f}{c+1} \right\rfloor - |I_{j+2}| \cdot f + c \\ & > (c+1) \cdot \left( \frac{|I_{j+2}| \cdot f}{c+1} - 1 \right) \\ & - |I_{j+2}| \cdot f + c = -1 \end{aligned}$$

Notice that for any two integers  $a$  and  $b$ , the inequality  $a - b > -1$  indicates  $a - b \geq 0$ . Thus, the above expression indicates that

$$Z_2 \geq Z_3 \quad (25)$$

Summarizing expressions (23), (24), and (25), we can see that for *Case 1* in which  $|O_{n-(j+1)}| - \lfloor |I_{j+2}| \cdot f / (c+1) \rfloor > 0$

and  $|I_{j+1}| \cdot f - \lfloor |O_{n-j}| / (c+1) \rfloor \leq |O_{n-(j+1)}|$ ,  $Z_{\text{even}}(c)$  is determined by

$$\begin{aligned} Z_{\text{even}}(c) &= \max\{Z_1, Z_2, Z_3\} = Z_2 \\ &= |O_{n-(j+1)}| \\ &+ \left\lfloor \frac{|I_{j+1}| \cdot f + |O_{n-(j+1)}| + \lfloor |I_{j+2}| \cdot f / (c+1) \rfloor}{c+2} \right\rfloor. \end{aligned} \quad (26)$$

**Case 2:**  $|O_{n-(j+1)}| - \lfloor |I_{j+2}| \cdot f / (c+1) \rfloor > 0$  and  $|I_{j+1}| \cdot f - \lfloor |O_{n-j}| / (c+1) \rfloor > |O_{n-(j+1)}|$

For the network illustrated in Fig. 4, one configuration corresponding to this case 2 is when  $c = 4$  and  $f = 6$ , still  $j = \lceil n - \log_2 f \rceil / 2 - 1 = 1$ . Thus, we have  $|O_{n-(j+1)}| - \lfloor |I_{j+2}| \cdot f / (c+1) \rfloor = |O_4| - \lfloor |I_3| \cdot 6 / (4+1) \rfloor = 8 - 4 > 0$  and  $|I_{j+1}| \cdot f - \lfloor |O_{n-j}| / (c+1) \rfloor = |I_2| \cdot 6 - \lfloor |O_5| / (4+1) \rfloor = 9 > 8 = |O_{n-(j+1)}|$ . For this *Case 2*, inequality (21) may hold but inequality (20) never holds due the fact that  $0 \leq l \leq |O_{n-(j+1)}|$ . Thus, we always have

$$|I_{j+1}| \cdot f - l \geq \left\lfloor \frac{|I_{j+1}| \cdot f - l + |O_{n-j}|}{c+2} \right\rfloor$$

By separating the whole range  $0 \leq l \leq |O_{n-(j+1)}|$  for  $l$  into two subranges  $|O_{n-(j+1)}| \geq l \geq |O_{n-(j+1)}| - \lfloor |I_{j+2}| \cdot f / (c+1) \rfloor$  (i.e.,  $8 \geq l \geq 4$  for the above configuration) and  $|O_{n-(j+1)}| - \lfloor |I_{j+2}| \cdot f / (c+1) \rfloor > l \geq 0$  (i.e.,  $4 > l \geq 0$  for the same configuration), we can prove in a similar way as that of *Case 1* that  $Z_{\text{even}}(c)$  for *Case 2* is still determined by the formula (26).

**Case 3:**  $|O_{n-(j+1)}| - \lfloor |I_{j+2}| \cdot f / (c+1) \rfloor \leq 0$  and  $|I_{j+1}| \cdot f - \lfloor |O_{n-j}| / (c+1) \rfloor \leq |O_{n-(j+1)}|$

For the network illustrated in Fig. 4, one configuration corresponding to this *Case 3* is when  $c = 1$  and  $f = 7$ , so we still have  $j = \lceil n - \log_2 f \rceil / 2 - 1 = 1$ . Thus,  $|O_{n-(j+1)}| - \lfloor |I_{j+2}| \cdot f / (c+1) \rfloor = |O_4| - \lfloor |I_3| \cdot 7 / (1+1) \rfloor = 8 - 14 \leq 0$  and  $|I_{j+1}| \cdot f - \lfloor |O_{n-j}| / (c+1) \rfloor = |I_2| \cdot 7 - \lfloor |O_5| / (1+1) \rfloor = 6 \leq 8 = |O_{n-(j+1)}|$ . For this *Case 3*, inequality (20) may hold depending on the value of  $l$  but inequality (21) always holds due to the fact that  $0 \leq l \leq |O_{n-(j+1)}|$ . Thus, we only need to separate the whole range  $0 \leq l \leq |O_{n-(j+1)}|$  for  $l$  into the following two subranges for the evaluation of  $Z_{\text{even}}(c)$ .

$$\begin{aligned} Z_3 &= \max_{0 \leq l < |O_{n-(j+1)}| - \lfloor |I_{j+2}| \cdot f / (c+1) \rfloor} \left\{ \left\lfloor \frac{|I_{j+1}| \cdot f + |I_{j+2}| \cdot f + |O_{n-j}| + |O_{n-(j+1)}| + c \cdot l}{c+2} \right\rfloor \right\} \\ &= \left\lfloor \frac{|I_{j+1}| \cdot f + |I_{j+2}| \cdot f + |O_{n-j}| + |O_{n-(j+1)}| + c \cdot (|O_{n-(j+1)}| - \lfloor |I_{j+2}| \cdot f / (c+1) \rfloor - 1)}{c+2} \right\rfloor \\ &= |O_{n-(j+1)}| + \left\lfloor \frac{|I_{j+1}| \cdot f + |I_{j+2}| \cdot f + |O_{n-(j+1)}| - c \cdot \lfloor |I_{j+2}| \cdot f / (c+1) \rfloor - c}{c+2} \right\rfloor \end{aligned}$$

• If  $|O_{n-(j+1)}| \geq l \geq |I_{j+1}| \cdot f - \lfloor |O_{n-j}|/(c+1) \rfloor$  (i.e., the range  $8 \geq l \geq 6$  for the above configuration):

The value of  $\max_l(l + x_1 + x_2)$  in this subrange is just the  $Z_1$  determined by (22).

• If  $|I_{j+1}| \cdot f - \lfloor |O_{n-j}|/(c+1) \rfloor > l \geq 0$  (i.e., the range  $6 \geq l \geq 0$  for the above configuration):

When  $l$  falls within this subrange, inequality (20) does not hold. Thus, the value of  $\max_l(l + x_1 + x_2)$  in this subrange (denoted as  $Z_4$  here) is evaluated as

$$\begin{aligned} Z_4 &= \max_{0 \leq l < |I_{j+1}| \cdot f - \lfloor |O_{n-j}|/(c+1) \rfloor} \left\{ l \right. \\ &\quad \left. + \left\lfloor \frac{|I_{j+1}| \cdot f - l + |O_{n-j}|}{c+2} \right\rfloor \right. \\ &\quad \left. + (|O_{n-(j+1)}| - l) \right\} \\ &= |O_{n-(j+1)}| + \left\lfloor \frac{|I_{j+1}| \cdot f - 0 + |O_{n-j}|}{c+2} \right\rfloor \\ &= |O_{n-(j+1)}| + \left\lfloor \frac{|I_{j+1}| \cdot f + |O_{n-j}|}{c+2} \right\rfloor. \end{aligned} \quad (27)$$

Since  $c \geq 1$  indicates that

$$\frac{3}{2(c+2)} \geq \frac{1}{c+1}.$$

Thus, from  $|I_{j+1}| \cdot f \geq |O_{n-(j+1)}| = (1/2) |O_{n-j}|$  we know that

$$\begin{aligned} Z_4 &\geq |O_{n-(j+1)}| + \left\lfloor \frac{(1/2)|O_{n-j}| + |O_{n-j}|}{c+2} \right\rfloor \\ &= |O_{n-(j+1)}| + \left\lfloor \frac{3 \cdot |O_{n-j}|}{2 \cdot (c+2)} \right\rfloor \\ &\geq |O_{n-(j+1)}| \left\lfloor \frac{|O_{n-j}|}{c+1} \right\rfloor = Z_1. \end{aligned} \quad (28)$$

Summarizing expressions (27) and (28), we can see that for *Case 3* in which  $|O_{n-(j+1)}| - \lfloor |I_{j+2}| \cdot f/(c+1) \rfloor \leq 0$  and  $|I_{j+1}| \cdot f - \lfloor |O_{n-j}|/(c+1) \rfloor \leq |O_{n-(j+1)}|$ ,  $Z_{\text{even}}(c)$  is determined by

$$\begin{aligned} Z_{\text{even}}(c) &= \max\{Z_1, Z_4\} = Z_4 \\ &= |O_{n-(j+1)}| + \left\lfloor \frac{|I_{j+1}| \cdot f + |O_{n-j}|}{c+2} \right\rfloor. \end{aligned}$$

**Case 4:**  $|O_{n-(j+1)}| - \lfloor |I_{j+2}| \cdot f/(c+1) \rfloor \leq 0$  and  $|I_{j+1}| \cdot f - \lfloor |O_{n-j}|/(c+1) \rfloor > |O_{n-(j+1)}|$ .

For the network illustrated in Fig. 4, one configuration corresponding to the *Case 4* is when  $c = 2$  and  $f = 7$ , then  $j = \lceil n - \log_2 f \rceil / 2 - 1 = 1$ . Thus, we have  $|O_{n-(j+1)}| - \lfloor |I_{j+2}| \cdot f/(c+1) \rfloor = |O_4| - \lfloor |I_3| \cdot 7/(2+1) \rfloor = 8 - 9 \leq 0$  but  $|I_{j+1}| \cdot f - \lfloor |O_{n-j}|/(c+1) \rfloor = |I_2| \cdot 7 - \lfloor |O_5|/(2+1) \rfloor =$

$9 > 8 = |O_{n-(j+1)}|$ . For this case, inequality (20) never holds but inequality (21) always holds. Thus,  $Z_{\text{even}}(c)$  for *Case 4* can be simply determined as

$$\begin{aligned} Z_{\text{even}}(c) &= \max_{0 \leq l \leq |O_{n-(j+1)}|} \left\{ l \right. \\ &\quad \left. + \left\lfloor \frac{|I_{j+1}| \cdot f - l + |O_{n-j}|}{c+2} \right\rfloor \right. \\ &\quad \left. + (|O_{n-(j+1)}| - l) \right\} \\ &= |O_{n-(j+1)}| + \left\lfloor \frac{|I_{j+1}| \cdot f + |O_{n-j}|}{c+2} \right\rfloor \end{aligned}$$

which is the same as that of *Case 3*.

By summarizing all the above four cases together we conclude that when  $\lceil n - \log_2 f \rceil$  is even, the bound  $Z_{\text{even}}(c)$  on the maximum number of planes that the connections from  $I_{j+1}$  and the connections to  $O_{n-(j+1)}$  can jointly block is determined by

$$Z_{\text{even}}(c) = \begin{cases} |O_{n-(j+1)}| + \left\lfloor \frac{|I_{j+1}| \cdot f + |O_{n-j}|}{c+2} \right\rfloor, & \text{if } |O_{n-(j+1)}| - \left\lfloor \frac{|I_{j+2}| \cdot f}{c+1} \right\rfloor \leq 0 \\ |O_{n-(j+1)}| + \left\lfloor \frac{|I_{j+1}| \cdot f + |O_{n-(j+1)}| + \lfloor |I_{j+2}| \cdot f/(c+1) \rfloor}{c+2} \right\rfloor, & \text{if } |O_{n-(j+1)}| - \left\lfloor \frac{|I_{j+2}| \cdot f}{c+1} \right\rfloor > 0. \end{cases} \quad (29)$$

If we define  $m_1 = |I_{j+1}|f$ ,  $m_2 = |O_{n-j-1}|$  and  $m_{\text{even}} = \sum_{i=1}^j |I_i| \cdot f + \sum_{k=1}^{n-(j+1)} |O_k| + 1$  for the case that  $\lceil n - \log_2 f \rceil$  is even, we can get the expressions (9)–(10) immediately based on the formula (29), the condition (19) and the equation  $j = \lceil n - \log_2 f \rceil / 2 - 1$  for  $j$ .

2) When  $\lceil n - \log_2 f \rceil$  is odd:

For the case that  $\lceil n - \log_2 f \rceil$  is odd, we know from the Proof of Theorem 1 that we can also determine a unique integer  $j$  by  $j = (\lceil n - \log_2 f \rceil - 1)/2$ , which indicates that  $2^{n-2j-1} \leq f < 2^{n-2j}$  or  $|O_{n-j}| \leq |I_{j+1}|f < |O_{n-j+1}|$  (we can also refer to Fig. 4, except that  $|O_{n-j}| \leq |I_{j+1}|f < |O_{n-j+1}|$  now). Based on the conditions (7) for  $c = 0$  and (8) for  $c = n$ , we can see that the connections from  $\bigcup_{i=1}^j I_i$  and the connections to  $\bigcup_{k=1}^{n-(j+2)} O_k$  can always block  $\sum_{i=1}^j |I_i| \cdot f + \sum_{k=1}^{n-(j+2)} |O_k|$  planes, regardless of the blocking type and parameter  $c$ . Again, the difference between the two conditions is determined by the connections from  $I_{j+1}$  and connections to  $O_{n-(j+1)}$ . When  $c = n$ , only link-blocking can occur and the connections from  $I_{j+1}$  must be destined for  $O_{n-(j+1)}$  to create link-blocking for the tagged path. Thus, the total number of planes required for nonblocking is

$$\begin{aligned} &\sum_{i=1}^j |I_i| \cdot f + \sum_{k=1}^{n-(j+2)} |O_k| \\ &\quad + \min(|I_{j+1}| \cdot f, |O_{n-(j+1)}|) + 1 \\ &= \sum_{i=1}^j |I_i| \cdot f + \sum_{k=1}^{n-(j+1)} |O_k| + 1. \end{aligned} \quad (30)$$

For the case that  $c = 0$ , the connections from  $I_{j+1}$  to  $O_{n-j}$  can block  $\min(|I_{j+1}|f, |O_{n-j}|) = |O_{n-j}|$  planes and the connections from  $I_{j+2}$  to  $O_{n-(j+1)}$  can block  $\min(|I_{j+2}|f, |O_{n-(j+1)}|) = |O_{n-(j+1)}|$  planes. So the total number of planes required for nonblocking now becomes

$$\begin{aligned} & \sum_{i=1}^j |I_i| \cdot f + \sum_{k=1}^{n-(j+2)} |O_k| \\ & + \min(|I_{j+1}|f, |O_{n-j}|) \\ & + \min(|I_{j+2}|f, |O_{n-(j+1)}|) + 1 \\ & = \sum_{i=1}^j |I_i| \cdot f + \sum_{k=1}^{n-j} |O_k| + 1. \end{aligned} \quad (31)$$

For the general case that  $0 < c < n$ , we shall determine the maximum number of planes that the connections from  $I_{j+1}$  and the connections to  $O_{n-(j+1)}$  can jointly block based on the combination of link-blocking and crosstalk-blocking. Define the parameters  $l, x_1$  and  $x_2$  in the same way as that of the case when  $\lceil n - \log_2 f \rceil$  is even (please refer to Fig. 4), then an upper bound  $Z_{\text{odd}}(c)$  on the maximum number of planes that the connections from  $I_{j+1}$  and the connections to  $O_{n-(j+1)}$  can jointly block is determined by

$$Z_{\text{odd}}(c) = \max_l(l + x_1 + x_2) \quad (32)$$

Subject to :

$$0 \leq l \leq |O_{n-(j+1)}| \quad (33)$$

$$0 \leq x_1 \leq \min\left(|I_{j+1}| \cdot f - l, |O_{n-j}|, \left\lfloor \frac{|I_{j+1}| \cdot f - l + |O_{n-j}|}{c+2} \right\rfloor\right) \quad (34)$$

$$0 \leq x_2 \leq \min\left(|O_{n-(j+1)}| - l, \left\lfloor \frac{|I_{j+2}| \cdot f + |O_{n-(j+1)}| - l}{c+2} \right\rfloor\right). \quad (35)$$

The constraint (34), which is different from its counterpart in (17), is due to the facts that  $|O_{n-j}| \leq |I_{j+1}|f$  and the term  $|I_{j+1}|f - l$  may be smaller than  $|O_{n-j}|$ . Then an  $f$ -cast  $\log_2(2^n, 0, K, c)$  network with a general  $c(0 < c < n)$  and odd  $\lceil n - \log_2 f \rceil$  is SNB if

$$K \geq \sum_{i=1}^j |I_i| \cdot f + \sum_{k=1}^{n-(j+2)} |O_k| + Z_{\text{odd}}(c) + 1. \quad (36)$$

We can also prove that  $Z_{\text{odd}}(c)$  in (32) satisfies the following constraints:

$$|O_{n-(j+1)}| \leq Z_{\text{odd}}(c) \leq |O_{n-(j+1)}| + |O_{n-j}|.$$

Thus, the condition  $\sum_{i=1}^j |I_i| \cdot f + \sum_{k=1}^{n-(j+2)} |O_k| + Z_{\text{even}}(c) + 1$  in (36) for general  $c$  is also lower-bounded by the condition (30) for  $c = n$  and upper-bounded by the condition (31) for  $c = 0$ .

Since  $c \geq 1$  indicates that

$$2 \geq 1 + \frac{2}{c+1}.$$

Thus, from the condition  $|O_{n-j+1}| > |I_{j+1}|f \geq |O_{n-j}| = 2|O_{n-(j+1)}|$  for the case  $\lceil n - \log_2 f \rceil$  is odd we know that

$$\begin{aligned} |I_{j+1}| \cdot f & \geq \left(1 + \frac{2}{c+1}\right) \cdot |O_{n-(j+1)}| \\ & \geq |O_{n-(j+1)}| + \left\lfloor \frac{|O_{n-j}|}{c+1} \right\rfloor \end{aligned}$$

or

$$|I_{j+1}| \cdot f - \left\lfloor \frac{|O_{n-j}|}{c+1} \right\rfloor \geq |O_{n-(j+1)}|. \quad (37)$$

Notice that we always have  $|O_{n-(j+1)}| \geq l \geq 0$ , so expressions (37) and (20) imply that

$$|I_{j+1}| \cdot f - l \geq \left\lfloor \frac{|I_{j+1}| \cdot f - l + |O_{n-j}|}{c+2} \right\rfloor.$$

Notice also that

$$\begin{aligned} |O_{n-j}| & \leq \left\lfloor \frac{|I_{j+1}| \cdot f - l + |O_{n-j}|}{c+2} \right\rfloor l \\ & \Leftrightarrow \leq |I_{j+1}| \cdot f - (c+1) \cdot |O_{n-j}|. \end{aligned} \quad (38)$$

Again, from  $c \geq 1$  and  $|O_{n-j+1}| > |I_{j+1}|f$  we know that

$$\begin{aligned} |I_{j+1}| \cdot f - (c+1)|O_{n-j}| & \leq |I_{j+1}| \cdot f - 2|O_{n-j}| \\ & = |I_{j+1}| \cdot f - |O_{n-j+1}| < 0. \end{aligned}$$

Thus, the condition (38) never holds and we always have

$$|O_{n-j}| \geq \left\lfloor \frac{|I_{j+1}| \cdot f - l + |O_{n-j}|}{c+2} \right\rfloor. \quad (39)$$

Based on the results (37) and (39), the optimization problem (32)–(35) can be simplified as

$$Z_{\text{odd}}(c) = \max_l(l + x_1 + x_2)$$

Subject to :  $0 \leq l \leq |O_{n-(j+1)}|$

$$0 \leq x_1 \leq \left\lfloor \frac{|I_{j+1}| \cdot f - l + |O_{n-j}|}{c+2} \right\rfloor$$

$$\begin{aligned} 0 \leq x_2 & \leq \min\left(|O_{n-(j+1)}| \right. \\ & \left. - l, \left\lfloor \frac{|I_{j+2}| \cdot f + |O_{n-(j+1)}| - l}{c+2} \right\rfloor\right). \end{aligned} \quad (40)$$

Based on an approach similar to that of the case when  $\lceil n - \log_2 f \rceil$  is even, we can prove that  $Z_{\text{odd}}(c)$  in (40) is given by

$$Z_{\text{odd}}(c) = \begin{cases} |O_{n-(j+1)}| + \left\lfloor \frac{|I_{j+1}| \cdot f + |O_{n-j}|}{c+2} \right\rfloor, & \text{if } |O_{n-(j+1)}| - \left\lfloor \frac{|I_{j+2}| \cdot f}{c+1} \right\rfloor \leq 0 \\ |O_{n-(j+1)}| + \left\lfloor \frac{|I_{j+1}| \cdot f + |O_{n-(j+1)}| + \lfloor |I_{j+2}| \cdot f / (c+1) \rfloor}{c+2} \right\rfloor, & \text{if } |O_{n-(j+1)}| - \left\lfloor \frac{|I_{j+2}| \cdot f}{c+1} \right\rfloor > 0. \end{cases} \quad (41)$$

By determining  $j$  by the formula  $j = (\lceil n - \log_2 f \rceil - 1)/2$ , defining  $m_3 = |I_{j+1}|f$ ,  $m_4 = |O_{n-j-1}|$  and  $m_{\text{odd}} =$

$\sum_{i=1}^j |I_i| \cdot f + \sum_{k=1}^{n-(j+1)} |O_k| + 1$  for the case that  $\lceil n - \log_2 f \rceil$  is odd, we can get the expressions (11) and (12) based on formula (41) and condition (36). QED.

If we set  $f = 2^n - 1$  (multicast) in Theorem 3, then  $\lceil n - \log_2 f \rceil = 1$  is odd and  $\lfloor n + \log_2 f \rfloor = 2n - 1$ , so we can get the following nonblocking condition for a multicast  $\log_2(2^n, 0, K, c)$  network based on formulas (11) and (12).

*Corollary 3.5:* A  $\log_2(2^n, 0, K, c)$  network is multicast SNB if

$$K \geq \begin{cases} 2^{n-1} + \left\lfloor \frac{2^n - 1 + 2^{n-1}}{c+2} \right\rfloor, \\ \quad \text{if } 0 < c \leq \lfloor 7 - 2^{3-n} \rfloor \\ 2^{n-1} + \left\lfloor \frac{2^n - 1 + 2^{n-2} + \lfloor 2(2^n - 1)/(c+1) \rfloor}{c+2} \right\rfloor, \\ \quad \text{if } \lfloor 7 - 2^{3-n} \rfloor < c < n. \end{cases}$$

When we set  $f = 1$ , we can prove that our conditions in Theorem 3 reduce to the following conditions for SNB unicast  $\log_2(2^n, 0, K, c)$  networks.

*Corollary 3.6:* When  $f = 1$ , the conditions in (9) and (11) become the following: When  $\lceil n - \log_2 f \rceil = n$  is even:

$$K \geq (3/2)\sqrt{2^n} - 1 + \left\lfloor \frac{\sqrt{2^n}}{c+1} \right\rfloor, \quad 0 < c < n.$$

When  $\lceil n - \log_2 f \rceil = n$  is odd, the condition becomes

$$K \geq \begin{cases} \sqrt{2^{n+1}} - 1 + \left\lfloor \frac{\sqrt{2^{n+1}}}{c+2} \right\rfloor, \\ \quad \text{if } 0 < c \leq 3 \\ \sqrt{2^{n+1}} - 1 + \left\lfloor \frac{(3/4)\sqrt{2^{n+1}} + \lfloor \sqrt{2^{n+1}}/(c+1) \rfloor}{c+2} \right\rfloor, \\ \quad \text{if } 3 < c < n. \end{cases}$$

*Remark 3.1:* Corollary 3.6 indicates that for the special unicast  $\log_2(2^n, 0, K, c)$  networks with  $f = 1$ , our bounds of nonblocking conditions are as tight as that of the bounds developed in [7] for a smaller value of  $c$  (i.e., when  $c \leq n/2$  for the case  $n$  even and when  $c \leq 3$  for the case  $n$  odd) but are slightly less tighter than the bounds developed in [7] when  $c$  takes a larger value<sup>7</sup>.

*Remark 3.2:* In the Proof of Theorem 3, we assumed that each connection from  $\bigcup_{i=1}^j I_i$  and each connection to  $\bigcup_{k=1}^{n-(j+2)} O_k$  can always block a distinct plane, regardless of the blocking type (link-blocking or crosstalk-blocking) and parameter  $c$ . Notice that the sources and destinations of above connection may be within the sets  $I_{j+1}$  and  $O_{n-(j+1)}$ , so we may have less connections remaining in  $I_{j+1}$  and  $O_{n-(j+1)}$  to create additional blockings. If we consider all these possibilities in the formulations (15)–(18) and (32)–(35), we may get a tighter bound. But the problem is that there are too many details to consider, and also we may not be able to get a close-form formulation bound as we did now if the formulations (15)–(18) and (32)–(35) become too complex. It can be a future research topic on how

<sup>7</sup>It is notable that a smaller value of  $c$  (or equivalently a stricter crosstalk constraint) is likely of more interest in practical applications due to the stringent bit-error rate requirements of optical flow transmission. Even for a larger value of  $c$ , our bound are slightly less tight than the bounds developed in [7]. E.g., for the larger size unicast  $\log_2(1024, 0, K, c)$  network with  $n = 10$  (even) and  $c > n/2 = 5$ , our bound requires at most four more planes than the bound in [7]; for the unicast  $\log_2(512, 0, K, c)$  network with  $n = 9$  (odd) and  $c \geq 5$ , our bound requires at most two more planes than the bound in [7].

to extend the optimization framework in the Proof of Theorem 3 to get a tighter bound (or ideally the tightest bound) on the nonblocking conditions of general  $f$ -cast  $\log_2(2^n, 0, K, c)$  networks.

#### IV. NETWORKS WITH EXTRA STAGES

For a network with extra stages, each plane has multiple paths between each input-output pair and a connection is blocked in the plane only if all its paths in the plane are blocked (please refer to Fig. 2).

##### A. Strictly Nonblocking $f$ -Cast $\log_2(2^n, m, K, 0)$ Networks

For a  $\log_2(2^n, m, K, 0)$  network with crosstalk-free constraint ( $c = 0$ ), we will focus on the tagged SEs and consider only the crosstalk blocking (node blocking) in our analysis.

*Theorem 4:* A  $\log_2(2^n, m, K, 0)$  network is  $f$ -cast SNB iff

$$K > \begin{cases} N - 1, & \text{if } f \geq N - 2, \\ (f + 1) \left( \left\lfloor \log_2 \frac{N+f}{f+1} \right\rfloor - 1 \right) \\ \quad + \frac{N-1 - \left( 2^{\left\lfloor \log_2 \frac{N+f}{f+1} \right\rfloor - 1} - 1 \right) (f+1)}{2^{\left\lfloor \log_2 \frac{N+f}{f+1} \right\rfloor - 1}}, \\ \quad \text{if } N - 2 > f \geq \frac{N-2^{m+1}}{2^{m+1}-1}, \\ (f + 1)(m + 1) + \frac{N-1 - (2^{m+1}-1)(f+1)}{2^m}, \\ \quad \text{if } \frac{N-2^{m+1}}{2^{m+1}-1} > f \geq \frac{N-2^{n-1}}{2^{n-1}-1}, \\ (f + 1)(m + 1) + 2^{n-m-1} - 2, \\ \quad \text{if } \frac{N-2^{n-1}}{2^{n-1}-1} > f \geq 2^{n-m-2}, \\ (f + 1) \cdot m + \left( 2^{\left\lfloor \frac{n-m-\log_2 f}{2} \right\rfloor - 1} - 1 \right) \\ \quad \cdot f + 2^{\left\lfloor \frac{n-m+\log_2 f}{2} \right\rfloor} - 1 \\ \quad + \min \left( 2^{\left\lfloor \frac{n-m+\log_2 f}{2} \right\rfloor}, 2^{\left\lfloor \frac{n-m-\log_2 f}{2} \right\rfloor - 1} \cdot f \right), \\ \quad \text{if } 2^{n-m-2} > f \geq 1. \end{cases} \quad (42)$$

*Proof:* This theorem will be proved by extending the Hwang's arguments for electronic networks with only link-blocking [20] and Lea's arguments for the photonic ones with only unicast traffic ( $f = 1$ ) [7].

1)  $f \geq N - 2$ : When  $f \geq N - 2$ , we have  $|I_1|f + |O_1| \geq 1 \cdot N - 2 + 1 = N - 1$ , so the connections from  $I_1$  and  $O_1$  may use up all the output ports. Notice that under the crosstalk-free constraint, each connection from  $I_1$  or to  $O_1$  can block one plane, thus the total number of blocked planes is just  $N - 1$ .

2)  $N - 2 > f \geq \frac{N-2^{m+1}}{2^{m+1}-1}$ : When  $f$  falls within this range, we can determine a unique integer  $k$ , following a reasoning similar to that Theorem 1 proof, such that

$$\frac{N - 2^k}{2^k - 1} > f \geq \frac{N - 2^{k+1}}{2^{k+1} - 1}, 1 \leq k \leq m. \quad (43)$$

Since

$$\sum_{i=1}^k (|I_i| \cdot f + |O_i|) = (2^k - 1)(f + 1)$$

So expression (43) indicates that  $\sum_{i=1}^k (|I_i| \cdot f + |O_i|) < N - 1$  but  $\sum_{i=1}^{k+1} (|I_i| \cdot f + |O_i|) \geq N - 1$ . Thus, the connections from  $\bigcup_{i=1}^k I_i$  and the connections to  $\bigcup_{i=1}^k O_i$  cannot use up all the outputs, but the connections from  $\bigcup_{i=1}^{k+1} I_i$  and the connections to  $\bigcup_{i=1}^{k+1} O_i$  can use up all the outputs. Notice that under the crosstalk-free constraint, we need  $2^{i-1}$  connections from  $I_i$  or to  $O_i$  to block one plane for  $1 \leq i \leq k + 1$ , so the total number of blocked planes will be

$$\begin{aligned} & \sum_{i=1}^k \frac{(|I_i| \cdot f + |O_i|)}{2^{i-1}} \\ & + \frac{N - 1 - \sum_{i=1}^k (|I_i| \cdot f + |O_i|)}{2^{(k+1)-1}} \\ & = (f + 1)k + \frac{N - 1 - (2^k - 1)(f + 1)}{2^k}. \end{aligned} \quad (44)$$

It is notable that (43) also indicates that

$$\log_2 \frac{N + f}{f + 1} - 1 \leq k < \log_2 \frac{N + f}{f + 1}.$$

Since that

$$k = \left\lceil \log_2 \frac{N + f}{f + 1} - 1 \right\rceil = \left\lfloor \log_2 \frac{N + f}{f + 1} \right\rfloor - 1. \quad (45)$$

Substituting (45) into (44) will result in the condition in (42) corresponding to the case that  $N - 2 > f \geq (N - 2^{m+1}) / (2^{m+1} - 1)$ .

3)  $\frac{N - 2^{m+1}}{2^{m+1} - 1} > f \geq \frac{N - 2^{n-1}}{2^{n-1} - 1}$ : From the above analysis, we know that the current upper bound indicates that the connections from  $\bigcup_{i=1}^{m+1} I_i$  and connections to  $\bigcup_{i=1}^{m+1} O_i$  cannot use up all the outputs. On the other hand, the lower bound implies

$$\begin{aligned} & |I_{m+2}| \cdot f + \sum_{i=1}^{m+1} (|I_i| \cdot f + |O_i|) \\ & = 2^{m+1} \cdot f + (2^{m+1} - 1)(f + 1) \\ & \geq 2^{m+1} \cdot \frac{N - 2^{n-1}}{2^{m+1} - 1} + N - 2^{n-1} + 2^{m+1} - 1 \\ & > N + 2^{m+1} - 1 > N - 1 \end{aligned}$$

and

$$\sum_{i=1}^{m+1} |I_i| \cdot f \geq N - 2^{n-1} = O_n.$$

Thus, the connections from  $\bigcup_{i=1}^{m+2} I_i$  and connections to  $\bigcup_{i=1}^{m+1} O_i$  now can use up all the outputs, and the connections from  $\bigcup_{i=1}^{m+1} I_i$  can use up all the outputs in  $O_n$ .<sup>8</sup> Notice that under the crosstalk-free constraint, we need  $2^{i-1}$  connections from  $I_i$  or to  $O_i$  to block one plane for  $1 \leq i \leq m + 1$ . For  $i > m + 1$ , we always need  $2^m$  connections from  $I_i$  or to  $O_i$  to

<sup>8</sup>Notice that under the crosstalk-free constraint, the intersecting paths from  $I_i, i = 1, \dots, m + 1$ , can reach any outputs but the intersecting paths from  $I_{m+2}$  can only arrive at the outputs within  $\bigcup_{i=1}^{n-1} O_i$ .

block one plane. Thus, the total number of blocked planes now becomes

$$\begin{aligned} & \sum_{i=1}^{m+1} \frac{(|I_i| \cdot f + |O_i|)}{2^{i-1}} \\ & + \frac{N - 1 - \sum_{i=1}^{m+1} (|I_i| \cdot f + |O_i|)}{2^m} \\ & = (f + 1)(m + 1) + \frac{N - 1 - (2^{m+1} - 1)(f + 1)}{2^m}. \end{aligned}$$

4)  $\frac{N - 2^{n-1}}{2^{m+1} - 1} > f \geq 2^{n-m-2}$ : The upper bound here indicates that  $\sum_{i=1}^{m+1} |I_i| \cdot f < N - 2^{n-1} = O_n$ , so the connections from  $\bigcup_{i=1}^{m+1} I_i$  cannot use up all the outputs in  $O_n$ . The lower bound implies

$$|I_{m+2}| \cdot f \geq 2^{m+1} \cdot 2^{n-m-2} > \sum_{i=1}^{n-1} O_i.$$

Thus, the connections from  $I_{m+2}$  can use up all the outputs remaining in  $\bigcup_{i=1}^{n-1} O_i$ . Notice that the intersecting paths from  $I_i, 1 \leq i \leq m + 1$ , can only arrive at the outputs within  $\bigcup_{i=1}^{n-1} O_i$ , so the total number of blocked planes is now given by

$$\begin{aligned} & \sum_{i=1}^{m+1} \frac{(|I_i| \cdot f + |O_i|)}{2^{i-1}} \\ & + \frac{\sum_{i=1}^{n-1} |O_i| - \sum_{i=1}^{m+1} |O_i|}{2^m} \\ & = (f + 1)(m + 1) + 2^{n-m-1} - 2. \end{aligned}$$

5)  $2^{n-m-2} > f \geq 1$ : From above analysis we know that when  $f < 2^{n-m-2}$ , the connections from  $\bigcup_{i=1}^m I_i$  and connections to  $\bigcup_{i=1}^m O_i$  can always block  $\sum_{i=1}^m (|I_i| \cdot f + |O_i|) / 2^{i-1} = (f + 1) \cdot m$  planes (due to the blocking in the first and last  $m$  stages), so we now focus on the blocking with the central  $n - m$  stages of the  $\log_2(n, m, K, 0)$  network.

Notice that for a plane of a  $\log_2(n, m, K, 0)$  network, its central  $n - m$  stages consist of  $2^m$  copies of a standard  $(n - m)$ -stage banyan network (please refer to Fig. 2). From the structure of  $\log_2(n, m, K, 0)$  network we can say that each IIS set of one such  $(n - m)$ -stage banyan network is just  $2^m$  times of its normal counterpart, and we need exactly  $2^m$  connections to block one plane of the  $\log_2(n, m, K, 0)$  network in its central  $n - m$  stages. Since we now have  $\sum_{i=1}^{m+1} |I_i| \cdot f < N - 2^{n-1} = O_n$ , and other outputs of these  $2^m$  smaller  $(n - m)$ -stage banyan networks are never used by connections from  $\bigcup_{i=1}^m I_i$ . Thus, the blocking analysis for the central  $n - m$  stages of the  $\log_2(n, m, K, 0)$  network is now equivalent to the blocking analysis of one  $\log_2(n - m, 0, K, 0)$  network without extra stages.

For the case that  $2^{n-m-2} > f \geq 1$ , following the same idea of the Proof of Theorem 1 we can determine an integer  $j$  by  $j = \lceil n - m - \log_2 f \rceil / 2 - 1$  when  $\lceil n - m - \log_2 f \rceil$  is even and  $2^{n-m-2j-2} \leq f < 2^{n-m-2j-1}$ , and determine an integer  $j$  by  $j = (\lceil n - m - \log_2 f \rceil - 1) / 2$  when  $\lceil n - m - \log_2 f \rceil$  is odd

and  $2^{n-m-2j-1} \leq f < 2^{n-m-2j}$ . In general, the parameter  $j$  for both cases can be calculated as

$$j = \lceil n - m - \log_2 f \rceil / 2 - 1. \quad (46)$$

Therefore, by replacing  $n$  with  $n - m$  and summarizing the (4), (7) and (8) together, we can say that the total number of planes blocked in the central  $n - m$  stages of a  $\log_2(n, m, K, 0)$  network is given by

$$(2^j - 1) \cdot f + 2^{n-m-j-1} - 1 + \min(2^{n-m-j-1}, 2^j \cdot f).$$

Thus, the total number of blocked planes now becomes

$$\begin{aligned} & (f + 1) \cdot m + (2^j - 1) \cdot f \\ & + 2^{n-m-j-1} - 1 \\ & + \min(2^{n-m-j-1}, 2^j \cdot f). \end{aligned} \quad (47)$$

Replacing  $j$  in (47) with the expression (46), the last condition in (42) follows.

The conditions in (42) are also necessary, because these maximum numbers of blocked planes can be achieved by simply creating a worst-case request pattern according to each case assumed in the proof. QED.

By setting  $f = 2^n - 1$  in Theorem 4, we can get the following nonblocking condition for a multicast  $\log_2(n, m, K, 0)$  network.

*Corollary 4.1:* A  $\log_2(n, m, K, 0)$  network is multicast SNB iff

$$K > 2^n - 1.$$

The Corollary 4.1 indicates that for a multicast SNB  $\log_2(n, m, K, 0)$  network, its nonblocking condition is actually independent of the parameter  $m$  and always same as that of its  $\log_2(n, 0, K, 0)$  counterpart.

The following Corollary indicates that when we set  $f = 1$ , our conditions in Theorem 4 reduce to the conditions developed in [7] for SNB unicast  $\log_2(n, m, K, 0)$  networks.

*Corollary 4.2:* When  $f = 1$ , the conditions for SNB  $\log_2(n, m, K, 0)$  network become the following:

$$K > \begin{cases} 2m + 2\sqrt{2^{n-m}} - 2, & \text{if } n - m \text{ is even,} \\ 2m + (3/2)\sqrt{2^{n-m+1}} - 2, & \text{if } n - m \text{ is odd.} \end{cases} \quad (48)$$

*Proof:* Theorem 4 says that when  $f = 1$ , the condition for a SNB  $\log_2(n, m, K, 0)$  network becomes

$$\begin{aligned} K & > 2m + 2^{\lceil \frac{n-m}{2} \rceil - 1} - 1 + 2^{\lfloor \frac{n-m}{2} \rfloor} - 1 \\ & + \min\left(2^{\lfloor \frac{n-m}{2} \rfloor}, 2^{\lceil \frac{n-m}{2} \rceil - 1}\right). \end{aligned}$$

Notice that when  $n - m$  is even,

$$\lceil (n - m)/2 \rceil = \lfloor (n - m)/2 \rfloor = (n - m)/2$$

and when  $n - m$  is odd,

$$\begin{aligned} \lceil (n - m)/2 \rceil & = (n - m + 1)/2, \\ \lfloor (n - m)/2 \rfloor & = (n - m - 1)/2. \end{aligned}$$

Summarizing the above three expressions together, we will get the condition in (48). QED.

### B. Strictly Nonblocking $f$ -Cast $\log_2(2^n, m, K, n+m)$ Networks

Based on a proof similar to that of Theorem 4, the following conditions for  $\log_2(n, m, K, n + m)$  networks with only link-blocking have been developed in [20].

*Theorem 5:* A  $\log_2(2^n, m, K, n + m)$  network is  $f$ -cast SNB iff

$$K > \begin{cases} \frac{N-1}{2}, & \text{if } f \geq N - 2, \\ \frac{(f+1)(\lceil \log_2 \frac{N+f}{f+1} \rceil - 1)}{2} \\ \quad + \frac{N-1 - \left(2^{\lceil \log_2 \frac{N+f}{f+1} \rceil - 1} - 1\right)(f+1)}{2^{\lceil \log_2 \frac{N+f}{f+1} \rceil}}, & \\ \quad \text{if } N - 2 > f \geq \frac{N-2^m}{2^m-1}, \\ \frac{(f+1)m}{2} + \frac{N-1 - (2^m-1)(f+1)}{2^m}, & \\ \quad \text{if } \frac{N-2^m}{2^m-1} > f \geq \frac{N-2^{n-1}}{2^m-1}, \\ \frac{(f+1)m}{2} + 2^{n-m-1} - 1, & \\ \quad \text{if } \frac{N-2^{n-1}}{2^m-1} > f \geq 2^{n-m-2}, \\ \frac{(f+1)m}{2} + \left(2^{\lceil \frac{n-m-\log_2 f}{2} \rceil - 1} - 1\right) \\ \quad \cdot f + 2^{\lfloor \frac{n-m+\log_2 f}{2} \rfloor} - 1, & \\ \quad \text{if } 2^{n-m-2} > f \geq 1. \end{cases}$$

By setting  $f = 1$  and  $f = N - 1$ , we have

*Corollary 4.3:* A  $\log_2(2^n, m, K, n + m)$  is unicast SNB iff

$$K > \begin{cases} m + (3/2)\sqrt{2^{n-m}} - 2, & \text{if } n - m \text{ is even,} \\ m + \sqrt{2^{n-m+1}} - 2, & \text{if } n - m \text{ is odd.} \end{cases}$$

*Corollary 4.4:* A  $\log_2(2^n, m, K, n + m)$  network is multicast SNB iff  $K > \lfloor (N - 1)/2 \rfloor = N/2 - 1$ .

*Remark 4.1:* For the most general  $f$ -cast  $\log_2(2^n, m, K, c)$  networks, the analysis of their nonblocking conditions becomes too complex for us to develop a formula by now.

## V. COMPARISONS

To illustrate the conditions developed in this paper, Table I shows the number of planes ( $K$ ) with the variations of network size ( $N$ ) and parameter  $f$  for  $f$ -cast  $\log_2(N, 0, K, 0)$  networks, Table II displays the required number of planes with different  $f$  and  $m$  values of  $f$ -cast  $\log_2(512, m, K, 0)$  networks (crosstalk-free) and of  $\log_2(2^n, m, K, n+m)$  networks (without constraints on crosstalk) for a comparison with F. K. Hwang's results [20], and Table III provides the number of planes of  $f$ -cast  $\log_2(1024, 0, K, c)$  networks as a function of crosstalk constraint ( $c$ ) and maximum number of connection outputs ( $f$ ).

All the three tables show that the number of planes required for a nonblocking  $f$ -cast network always grows monotonously as  $f$  increases. In particular, for an  $f$ -cast  $\log_2(N, 0, K, 0)$  network which has no extra stages but has the most strict crosstalk

TABLE I  
NUMBER OF PLANES ( $K$ ) FOR  $f$ -CAST  $\log_2(N, 0, K, 0)$  NETWORKS

$f \backslash N$	4	8	16	32	64	128	256	512	1024
1	3	5	7	11	15	23	31	47	63
2	4	6	10	14	22	30	46	62	94
3	4	7	11	17	25	37	53	77	109
4		8	12	20	28	44	60	92	124
5		8	13	21	31	47	67	99	139
6		8	14	22	34	50	74	106	154
7		8	15	23	37	53	81	113	169
8			16	24	40	56	88	120	184
9			16	25	41	59	91	127	191
10			16	26	42	62	94	134	198
11			16	27	43	65	97	141	205
12			16	28	44	68	100	148	212
13			16	29	45	71	103	155	219
14			16	30	46	74	106	162	226
15			16	31	47	77	109	169	233
16				32	48	80	112	176	240

TABLE II  
NUMBER OF PLANES FOR  $f$ -CAST  
 $\log_2(512, m, K, 0) / \log_2(512, m, K, n + m)$  NETWORKS

$f \backslash m$	0	1	2	3	4	5	6	7	8
1	47/31	33/24	27/17	21/14	19/11	17/10	17/9	17/9	17/9
2	62/46	49/31	36/25	31/18	26/16	25/13	24/13	23/12	23/12
3	77/53	57/39	45/29	37/23	33/19	31/17	29/16	29/15	29/15
4	92/60	65/46	54/33	43/27	40/22	37/20	34/19	34/17	34/17
5	99/67	73/50	59/37	49/30	45/25	41/23	39/21	39/20	39/20
6	106/74	81/53	64/41	55/32	50/28	45/25	44/23	44/22	44/22
7	113/81	89/57	69/45	61/35	55/31	49/28	49/25	49/25	49/25
8	120/88	97/60	74/49	67/37	60/34	53/30	53/27	53/27	53/27
9	127/91	101/64	79/51	71/40	63/36	57/32	57/29	57/29	57/29
10	134/94	105/67	84/53	75/42	66/38	61/33	61/31	61/31	61/31
11	141/97	109/71	89/55	79/45	69/40	65/35	65/33	65/33	65/33
12	148/100	113/74	94/57	83/47	72/42	69/36	69/35	69/35	69/35
13	155/103	117/78	99/59	87/50	75/44	73/38	73/37	73/37	73/37
14	162/106	121/81	104/61	91/52	78/46	77/39	77/39	77/39	77/39
15	169/109	125/85	109/63	95/55	81/48	81/41	81/41	81/41	81/41
16	176/112	129/88	114/65	99/57	85/50	85/43	85/43	85/43	85/43

TABLE III  
NUMBER OF PLANES FOR  $f$ -CAST  $\log_2(1024, 0, K, c)$  NETWORKS

$f \backslash c$	0	1	2	3	4	5	6	7	8	9	10
1	63	63	57	55	53	52	51	51	50	50	47
2	94	83	78	74	72	70	69	68	67	66	62
3	109	103	97	93	90	88	86	85	84	83	77
4	124	124	113	108	104	102	101	100	99	98	92
5	139	133	123	117	113	111	109	108	107	106	99
6	154	143	134	126	122	119	117	116	115	114	106
7	169	153	143	136	131	128	126	124	123	122	113
8	184	162	152	145	140	136	134	132	131	129	120
9	191	172	161	154	149	145	142	140	139	137	127
10	198	182	170	162	158	153	150	148	146	145	134
11	205	191	179	171	166	162	159	156	154	153	141
12	212	201	188	180	174	170	167	164	162	161	148
13	219	211	197	188	183	179	175	173	170	169	155
14	226	220	206	197	191	187	184	181	178	177	162
15	233	230	215	205	199	195	192	189	186	185	169
16	240	240	218	208	201	197	194	192	190	188	176

constraint ( $c = 0$ ), Table I shows that its number of planes will be the same as that of its multicast counterpart when  $f$  increases to only half of the network size  $N$ .

Table II indicates that the number of planes required for an  $f$ -cast  $\log_2(N, m, K, 0)$  or  $\log_2(2^n, m, K, n + m)$  network

with extra stages will not decrease anymore if the number of extra stages  $m$  is larger than a threshold, and the overall hardware cost of the network will actually increase with  $m$  after this threshold due to the more extra stages. We can also observe from Table II that such threshold for  $m$  decreases for both  $f$ -cast  $\log_2(N, m, K, 0)$  and  $\log_2(2^n, m, K, n + m)$  network with the increase of the value of  $f$ . For example, the threshold of  $m$  is 6 for the 3-cast  $\log_2(512, m, K, 0)$  network, but this threshold becomes 4 for its 13-cast counterpart. Table II also shows clearly that due to the strict crosstalk constraint, there is always a difference between the number of planes for an  $f$ -cast  $\log_2(N, m, K, 0)$  network and that for its  $\log_2(2^n, m, K, n + m)$  counterpart; such difference grows monotonously as the parameter  $f$  increases, whereas it decreases, albeit non-monotonously, as the parameter  $m$  grows until it becomes constant (beyond a given value of  $m$ ).

From Table III we can find that for an  $f$ -cast  $\log_2(N, 0, K, c)$  with a general crosstalk constraint  $c$ , although its number of planes decreases monotonously with the increase of  $c$ , this decrease in the number of planes is more significant when the value of  $c$  is smaller (e.g., less than 3 in Table III). For example, for the 4-cast  $\log_2(N, 0, K, c)$  network, its number of plane decreases from 124 to 108 when  $c$  increases from 0 to 3, but its number of plane decreases only from 108 to 100 when  $c$  increases further from 3 to 7. Thus, the results in Table III indicate that we can actually apply a more strict crosstalk constraint in the design of an  $f$ -cast  $\log_2(N, 0, K, c)$  without introducing a significant increase in hardware cost.

## VI. CONCLUSION

We have studied the design of strictly nonblocking  $f$ -cast photonic networks when the general banyan-based architecture is adopted and various crosstalk constraints are imposed. We proposed a novel optimization framework for determining the nonblocking conditions of  $f$ -cast photonic networks when a general crosstalk constraint is considered, and showed how to derive the close-form formulas for the nonblocking conditions under this framework. The results in this paper can help us to find the graceful tradeoff between crosstalk requirement and hardware cost in an  $f$ -cast photonic network, and we expect the methodology developed in this paper will also be useful for deriving the nonblocking conditions of other types of switching networks.

It is notable that in addition to the crosstalk issue addressed in this paper, other parameters like extinction ratios, added noise, required switching energy, etc. will also affect the final switch performance. How to extend the analysis in this paper to incorporate more performance metrics in the switch design can be an interesting future research direction. Notice also that we have only obtained a sufficient condition for a  $f$ -cast  $\log_2(2^n, 0, K, c)$  network, so another future research topic is how to extend the optimization framework proposed in this paper to get a tighter bound (or ideally the tightest bound, i.e., the sufficient and also necessary condition) for such a network. Finally, the nonblocking condition analysis of the general  $f$ -cast  $\log_2(2^n, m, K, c)$  networks remains to be explored further.

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