

Rearrangeable f -cast Multi- $\log_2 N$ Networks

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Abstract—Multi- $\log_2 N$ networks (or vertically stacked banyan networks) have been an attractive class of switching networks due to their small depth $O(\log N)$, absolute signal loss uniformity and good fault tolerance property. Recently, F.K.Hwang extended the study of multi- $\log_2 N$ networks to the general f -cast case, which covers the unicast case ($f = 1$) and multicast case ($f = N$) as special cases, and determined the conditions for these networks to be f -cast strictly nonblocking when the fan-out capability is available at both the input stage and middle banyan stage. In this paper, we study the rearrangeable f -cast multi- $\log_2 N$ networks under both node-blocking scenario (relevant to photonic switches) and link-blocking scenario (relevant to electronic switches). In particular, we consider the following three fan-out cases in our study: 1) no restriction on fan-out capability; 2) input stage has no fan-out capability; 3) middle banyan stage has no fan-out capability. We determine the necessary conditions for the first two cases while obtaining the necessary and also sufficient condition for the third one.

Index Terms— f -cast, multicast, multi- $\log_2 N$ switching networks, optical switches, rearrange.

I. INTRODUCTION

SWITCHING networks serve as the essential part of network routers and massive multiprocessor computing systems. Multi- $\log_2 N$ networks (or vertically stacked Banyan networks) have been an attractive class of switching networks for both electronic and photonic applications [1-17], because they have small depth $O(\log N)$, absolute signal loss uniformity and good fault tolerance property, and they also have the capability to guarantee a good crosstalk performance when they are adopted for constructing the photonic switches [2,7,9,12,16]. We can flexibly control the blocking property of a multi- $\log_2 N$ network by determining its number of stacked planes without sacrificing its loss uniformity property. Available results on the study of multi- $\log_2 N$ switching networks can be roughly divided into two categories: the results about nonblocking conditions, such as [1-15], and the results about blocking behavior analysis, such as [16,17]. The study of this paper falls within the first category.

To study a nonblocking network, we need to specify the connection requests property since it directly affects the

nonblocking condition of the network. The requests in a switching network can be one-to-one (unicast), one-to-many (multicast), one-to-all (broadcast), or more generally the f -cast, in which an input can simultaneously request up to f distinct outputs [18-21]. The multicast covers the broadcast as a special scenario, and the f -cast covers the unicast ($f = 1$) and multicast ($f = N$, i.e. f is unconstrained) as special cases. In general, we can define the *request session* of a network as the set of requests that originate from a common input and are destined for multiple distinct outputs, and we call a request session is *feasible* if its input and output(s) are all idle. The nonblocking networks can be *strictly nonblocking (SNB)*, *wide-sense nonblocking (WNB)*, *rearrangeably nonblocking (RNB)*, or *repackably nonblocking (RPNB)*. In a SNB network, we can always establish a feasible request session regardless of how other request sessions are established. Some results about SNB multi- $\log_2 N$ networks can be found in [1-5]. In a WNB network, we can establish a feasible request session without disturbing the existing ones if a special control algorithm is followed during the connection setup [5-10]. The current studies of WNB multicast networks focus mainly on the switch control algorithms based on the concept of blocking window, which was first proposed by Tscha and Lee [10] for the fixed window size case and was later extended by Kabacinski and Danilewicz [5] to the more general variable window size case. The RNB networks can also route any feasible request session, but one or more existing connections may have to be rerouted to establish the new session [11-14]. A RPNB network is similar to its RNB counterpart in the sense that the rerouting of existing connections is also required to guarantee the nonblocking property in such a network, but the difference is that the routing of existing connections in a RPNB network is done when an existing connection is deleted while in a RNB network such routing operation is conducted when a new request is blocked [22,18,15]. The RNB networks are attractive, because their hardware cost is usually lower than their SNB and WNB counterparts. Our interest of this paper is on the RNB f -cast multi- $\log_2 N$ networks.

The conditions for SNB f -cast Clos network and RNB f -cast Clos network have been addressed in [19], [21] and [20], respectively. Recently, F. K. Hwang extended the study of multi- $\log_2 N$ networks to the general f -cast case and determined the conditions for these networks to be f -cast strictly nonblocking when the fan-out capability is available at both the input stage and middle Banyan stage [18]. In this paper, we study the rearrangeable f -cast multi- $\log_2 N$ networks under both link-blocking scenario (relevant to electronic switches) and node-blocking scenario (relevant to photonic switches). Under the link-blocking constraint, only one

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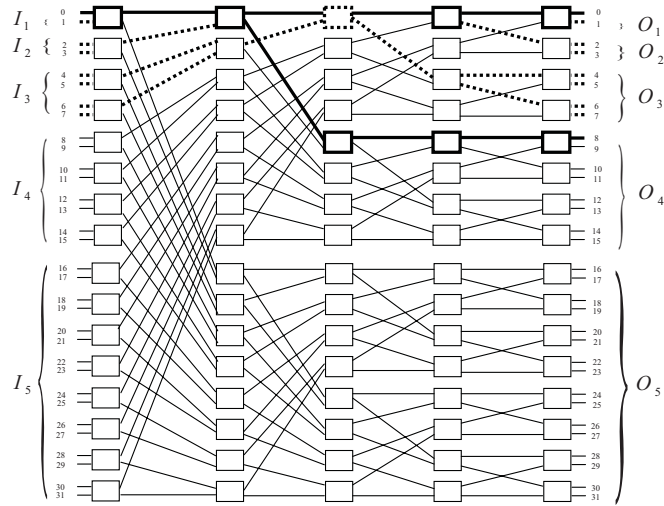


Fig. 1. A 32×32 Banyan network with its input intersecting sets and output intersecting sets (odd number of stages).

request session is allowed to use a link at one time, which is relevant to electronic switches [1,3-6,8,10,11,13,14]. On the other hand, the node-blocking constraint allows only one request session to use a switching element (SE) at one time. The node-blocking constraint is of more interest for photonic switches [2,7,9,12,16,17], where crosstalk happens when two signals pass through a common SE at the same time and the elimination of crosstalk is an important issue for making photonic switches work properly [26,27]. Similar to the fan-out restrictions of f -cast Clos networks [19,20], in this paper we also apply different fan-out restrictions to multi- $\log_2 N$ networks and study the networks without restriction on fan-out capability, the networks without input stage fan-out capability and networks without middle Banyan stage fan-out capability, respectively. We determine the necessary conditions for the first two cases while obtaining the necessary and also sufficient condition for the third one.

The paper is organized as follows. Section II introduces some basic definitions and notations to be used throughout this paper. Section III, IV and V present the conditions for the rearrangeable f -cast multi- $\log_2 N$ networks under above three fan-out cases, respectively. Section VI provides the comparison and discussion, and Finally, Section VII concludes the paper and discusses the future works.

II. DEFINITIONS

Banyan network [23] and its topological equivalence (e.g. baseline, omega) are a class of important switching networks and they are generally referred as Banyan networks [24]. A typical $N \times N$ Banyan network consists of $n = \log_2 N$ stages, each containing $N/2 \times 2 \times 2$ switching elements (SEs) and the link connections between adjacent stages are implemented by recursively applying the unshuffle interconnection pattern, as illustrated in Fig. 1.

Banyan networks have unique path for each input-output pair, which makes them blocking networks. Multi- $\log_2 N$ network is an interesting architecture for constructing Banyan-based nonblocking networks without sacrificing the loss uniformity property of Banyan networks. A multi- $\log_2 N$ network

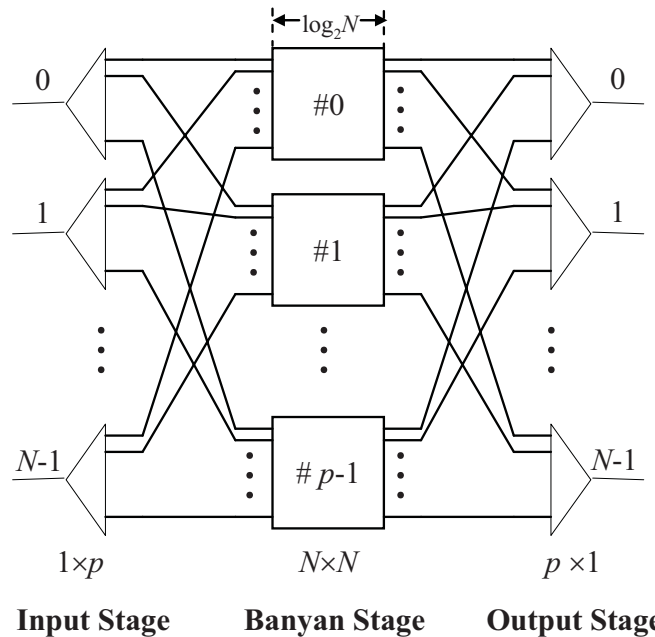


Fig. 2. Illustration of a multi- $\log_2 N$ network (vertically stacked Banyan network).

is constructed by vertically stacking p Banyan networks such that p paths between each input-output pair can be generated [1,25], as illustrated in Fig. 2. For a multi- $\log_2 N$ network, we define the input stage as the set of $N \times 1 \times p$ switches attached to the N input ports, the Banyan stage as the set of p equivalent $N \times N$ Banyan networks, and the output stage as the set of $N \times p \times 1$ switches attached to the N output ports.

Due to the topological symmetry architecture of a Banyan network, all paths in it have the same property in terms of blocking. Based on the methodology established in [2], we can conduct the blocking analysis of multi- $\log_2 N$ networks by focusing a tagged path and its associated input intersecting sets (IIS) and output intersecting sets (OIS). For a tagged path, all the SE's and links on the tagged path are called *tagged SE's* and *tagged links*, respectively. The stages of SE's and links are numbered from left (stage 1) to right (stage n). For the tagged path between the input 0 and output 0 (please refer to Fig. 1), the IIS $I_i = \{2^{i-1}, 2^{i-1} + 1, \dots, 2^i - 1\}$ is defined as the set of all inputs that intersect a tagged SE, for the first time, at stage i ; Symmetrically, the OIS $O_i = \{2^{i-1}, 2^{i-1} + 1, \dots, 2^i - 1\}$ is the set of all outputs that intersect a tagged SE at stage $n - i + 1$, $1 \leq i \leq n$.

For an f -cast multi- $\log_2 N$ network, we define a *request session* of the network as the set of requests that originate from a common input and are destined for at most f distinct outputs. We use the notation $\langle x, y \rangle$ to refer to a unicast session from input x to output y and use the notation $\langle x, (y, z) \rangle$ to represent a request session from input x to outputs y and z . We define further a *request frame* of a f -cast multi- $\log_2 N$ network as the set of all request sessions of the network and denote it as $\left(\begin{matrix} x_0, x_1, \dots, x_{N-1} \\ y_0, y_1, \dots, y_{N-1} \end{matrix} \right)$, where input x_i is mapped to output y_i , with $x_i, y_i \in \{0, 1, \dots, N-1\}$, $\{y_0, y_1, \dots, y_{N-1}\} = \{0, 1, N-1\}$, $x_0 \leq x_1 \leq \dots \leq x_{N-1}$ and a given input will appear maximum f times (f -cast).

An example of request frame is given in (1). Then a f -cast multi- $\log_2 N$ network is rearrangeably nonblocking (RNB) iff all request sessions of any request frame can be routed simultaneously.

Example 1: A request frame for 2-cast ($f = 2$) multi- $\log_2 16$ network (see top of next page).

In this paper, we study both the multi- $\log_2 N$ network with link-blocking constraint and the multi- $\log_2 N$ network with node-blocking constraint. Hereafter, we use notation $\log_2(f, N, p)$ to refer a f -cast multi- $\log_2 N$ network that has p planes (i.e., p vertically stacked copies of a Banyan network as illustrated in Fig. 2), and use notations $NB - \log_2(f, N, p)$ and $LB - \log_2(f, N, p)$ to refer to the $\log_2(f, N, p)$ networks with node-blocking constraint and link-blocking constraint, respectively.

Similar to that of the fan-out definitions for Clos network [19,20], we define that a $1 \times p$ switch or a SE in $\log_2(f, N, p)$ networks has the fan-out capability if any one-to-many mapping between its input(s) and outputs can be connected. We will say that the input stage (middle Banyan stage) of a $\log_2(f, N, p)$ network has the fan-out capability if each of its $1 \times p$ switch (SE) has the fan-out capability. In this paper, we will consider the following three fan-out cases in our study of $\log_2(f, N, p)$ networks:

- 1) *Full Fan-out:* both the input and Banyan stages have fan-out capability;
- 2) *Banyan Stage Fan-out:* only middle Banyan stage has fan-out capability;
- 3) *Input Stage Fan-out:* only input stage has fan-out capability.

We will determine the necessary conditions of rearrangeable $\log_2(f, N, p)$ networks for the above three fan-out cases and also prove that our necessary condition for the third case is also sufficient.

III. REARRANGEABLE $\log_2(F, N, P)$ NETWORKS WITH FULL FAN-OUT

For a $\log_2(f, N, p)$ network with full fan-out capability, the necessary conditions for the network to be RNB is the same as that of its unicast counterpart developed by C.-T.Lea [1], as summarized in the following theorem 1 and corollary 1.

Theorem 1: For a NB $\log_2(f, N, p)$ network with full fan-out and $1 \leq f \leq N$, a necessary condition for the network to be RNB is:

$$p \geq 2^{\lceil \frac{n}{2} \rceil} = \begin{cases} \sqrt{N} & \text{if } n \text{ is even} \\ \sqrt{2N} & \text{if } n \text{ is odd.} \end{cases} \quad (2)$$

where the symbol $\lceil x \rceil$ denotes the ceiling function of a variable x , which represents the least integer greater than or equal to x .

Proof: For a NB- $\log_2(f, N, p)$ network with node-blocking constraint, we only need to consider the node-blocking in our analysis. Based on the full fan-out assumption that the fan-out capability is available at both the input and Banyan stages, the requests of a request session may be routed independently through different planes. Therefore, we just need to focus on only one of these requests in our analysis and we regard the path of the selected request as the tagged path. Here, we focus on the tagged path between input 0 and output 0. Note that

the requests from the same session can not block each other, since they can share SEs. Thus, we only need to consider the requests from sessions other than that of the tagged path in our blocking analysis.

For the tagged SE at stage i , $1 \leq i \leq n$, the maximum number of distinct sessions that can intersect the tagged SE is given by $\min\left(\sum_{k=1}^i |I_k|, \sum_{k=1}^{n-i+1} |O_k|\right) = \min(2^i - 1, 2^{n-i+1} - 1)$. Note that all the distinct sessions passing through a common SE must fall within distinct planes to avoid node-blocking, but the requests from the same session may share a common SEs. Therefore, a necessary condition for a NB- $\log_2(f, N, p)$ network to be RNB is

$$\begin{aligned} p &\geq \max_{1 \leq i \leq n} \{ \min(2^i - 1, 2^{n-i+1} - 1) \} + 1 \\ &= \max_{1 \leq i \leq n} \{ \min(2^i - 1, 2^{n-i+1}) \} \\ &= 2^{\lceil \frac{n}{2} \rceil} = \begin{cases} \sqrt{N} & \text{if } n \text{ is even} \\ \sqrt{2N} & \text{if } n \text{ is odd.} \end{cases} \end{aligned}$$

It is notable that the above necessary condition of a NB- $\log_2(f, N, p)$ network is independent of the parameter f , this is due to the fact that this necessary condition is totally determined by the maximum number of distinct sessions that can intersect a tagged SE, which is only related to the size of input intersecting sets/output intersecting sets and thus is independent of parameter f . The independence with f of the above condition is illustrated in Fig. 3, where the Fig. 3(a) illustrates a scenario of $f = 2$ (with three multicast sessions $\langle 2, \{4, 5\} \rangle$, $\langle 3, \{6, 7\} \rangle$ and $\langle 4, \{2, 3\} \rangle$, and one unicast session $\langle 1, 1 \rangle$) and Fig. 3(b) illustrates a scenario of $f = 3$ (with two multicast sessions $\langle 1, \{1, 2, 3\} \rangle$ and $\langle 2, \{4, 5, 6\} \rangle$, and one unicast session $\langle 3, 7 \rangle$). Both scenarios in Fig. 3 result in a maximum number 3 of distinct intersecting sessions with a tagged SE (dashed SE), and this number is independent of parameter f . QED.

About the LB- $\log_2(f, N, p)$ with link-blocking constraint, we have the following condition that is same as that of its unicast counterpart developed by C.-T.Lea [1].

Corollary 1: For a LB- $\log_2(f, N, p)$ network with full fan-out and $1 \leq f \leq N$, a necessary condition for the network to be RNB is:

$$p \geq 2^{\lfloor \frac{n}{2} \rfloor} = \begin{cases} \sqrt{N}, & \text{if } n \text{ is even} \\ (1/2)\sqrt{2N}, & \text{if } n \text{ is odd.} \end{cases} \quad (3)$$

where the symbol $\lfloor x \rfloor$ denotes the floor function of a variable x , which represents the greatest integer less than or equal to x .

Proof: The proof of this Corollary is similar to that of the theorem 1 except that we will focus on the tagged links (rather than tagged SEs) and consider only the link blocking in our analysis. QED.

Remark 3.1: It is notable that for the NB- $\log_2(f, N, p)$ and LB- $\log_2(f, N, p)$ networks with only unicast requests ($f = 1$), the necessary conditions provided in Theorem 1 and Corollary 1 are also sufficient, see, for example, [1,11,12]. For the LB- $\log_2(f, N, p)$ networks with multiple-multicast connections ($f = N$), it has only been proved in [1] that the necessary condition in Corollary 1 is also sufficient for networks with $N = 4, 8$, and 16, respectively. For the general

$$\begin{pmatrix} 0 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 8 & 9 & 9 & 11 & 13 & 15 \\ 0 & 4 & 8 & 9 & 10 & 1 & 2 & 3 & 6 & 11 & 12 & 5 & 14 & 13 & 15 & 7 \end{pmatrix} \quad (1)$$

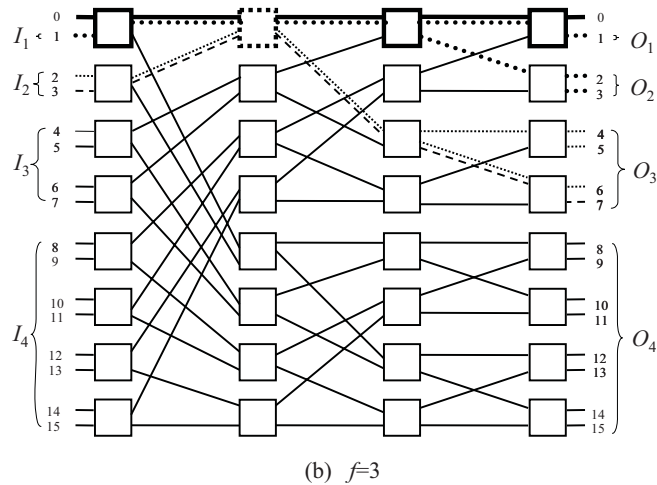
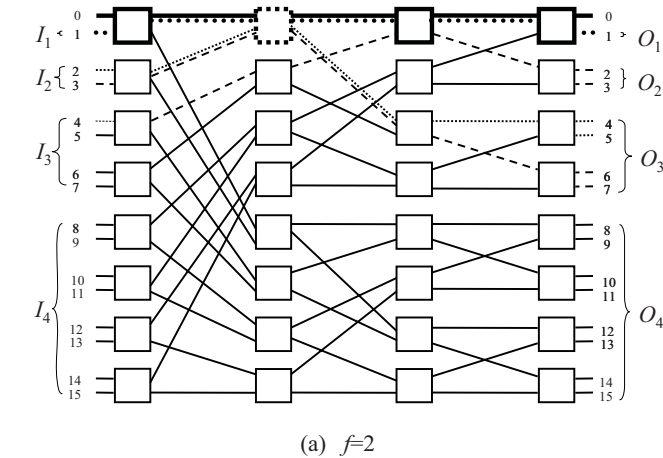


Fig. 3. Illustration of the worst case conflicting with the tagged path in a f -cast 16×16 banyan network. (a) A scenario for the case $f = 2$ with three multicast sessions $\langle 2, \{4, 5\} \rangle$, $\langle 3, \{6, 7\} \rangle$ and $\langle 4, \{2, 3\} \rangle$, and one unicast session $\langle 1, 1 \rangle$ (b) A scenario for the case $f = 3$ with two multicast sessions $\langle 1, \{1, 2, 3\} \rangle$ and $\langle 2, \{4, 5, 6\} \rangle$, and one unicast session $\langle 3, 7 \rangle$.

NB- $\log_2(f, N, p)$ and LB- $\log_2(f, N, p)$ networks with fan-out capability at both input and Banyan stages, it is still a challenging open problem to prove whether the conditions in Theorem 1 and Corollary 1 are also sufficient or not for these networks to be RNB, respectively. This remains to be deliberately explored further.

IV. REARRANGEABLE $\text{LOG}_2(F, N, P)$ NETWORKS WITH BANYAN STAGE FAN-OUT

For a $\log_2(f, N, p)$ network with fan-out capability only at its Banyan stage, all the requests of an f -cast session must be routed together (not independently) through a common plane. Before presenting the results for NB- $\log_2(f, N, p)$ and LB- $\log_2(f, N, p)$ networks under the new fan-out constraint, we first use the following example to illustrate the implication of this constraint upon the blocking properties.

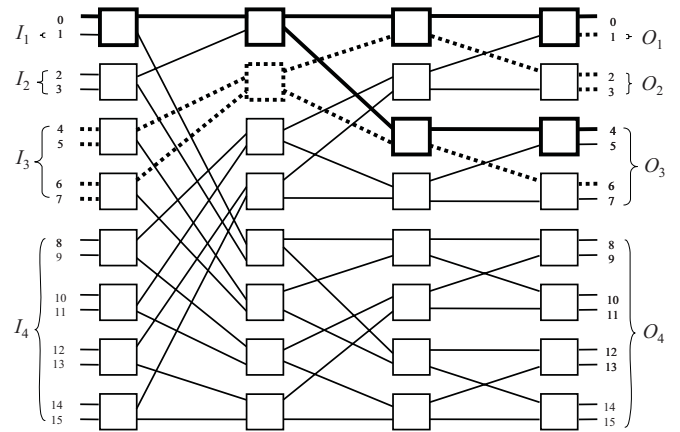


Fig. 4. Illustration of the blocking caused by the constraint of Banyan stage fan-out.

Example 2: Consider a NB- $\log_2(2, 16, p)$ network (Fig. 4) with the request frame shown in Example 1. We focus on the session $\langle 0, (0, 4) \rangle$ and regard its paths as the tagged paths (highlighted with bold lines). The paths of four connections that intersect the tagged paths, $\langle 4, 1 \rangle$, $\langle 5, 2 \rangle$, $\langle 6, 3 \rangle$ and $\langle 7, 6 \rangle$, are shown with the dashed lines. It is notable that in addition to the above four intersecting connections, we may have other connections that also intersect with the tagged paths, such as the connections $\langle 1, 5 \rangle$ and $\langle 2, 7 \rangle$ (or $\langle 3, 7 \rangle$). We focus on the four dashed intersecting connections, because they represent the maximum number of connections that conflict with the tagged session and also share a common SE (thus they can not share any plane under the node-blocking constraint).

The intersecting connections $\langle 4, 1 \rangle$, $\langle 5, 2 \rangle$ and $\langle 6, 3 \rangle$ will intersect the upper tagged path and the connection $\langle 7, 6 \rangle$ will conflict with lower tagged path. Since the above four intersecting connections all share the common dashed SE in Fig. 4, they must fall within four distinct planes to avoid node-blocking. Based on the constraint that the fan-out capability is available only at the Banyan stage (*Banyan Stage Fan-out*), the tagged session $\langle 0, (0, 4) \rangle$ must be routed together through a common plane. Since each of these four planes devoted to the four intersecting connections will block the tagged 2-cast session, we need one more plane to route the tagged session. Thus, for a NB- $\log_2(2, 16, p)$ network with Banyan stage fan-out, a necessary condition for it to be RNB is $p \geq \sqrt{16} + 1 = 5$ rather than $p \geq \sqrt{16} = 4$ as that of its full fan-out counterpart. Actually, it is easy to verify that for this example if fan-out is also available at the input stage (full fan-out case), then the necessary condition will become $p \geq 4$ again, because we are able to route the two requests of the tagged session $\langle 0, (0, 4) \rangle$ without blocking among two of these four planes devoted to the four intersecting connections.

Example 2 indicates that for networks with Banyan stage fan-out, we need to determine a new necessary condition different from that of its full fan-out counterpart, as summarized

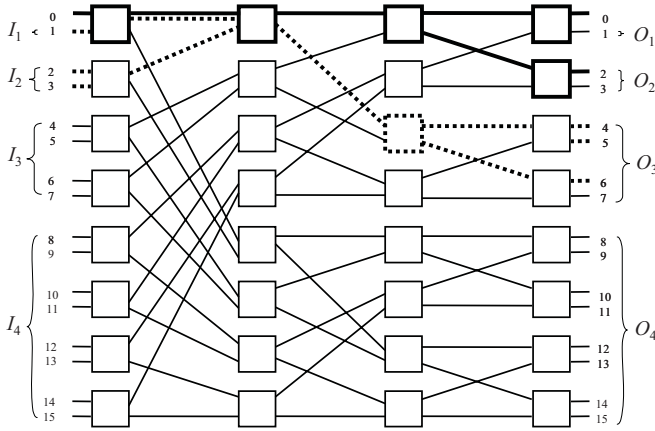


Fig. 5. An example for the case that the output i of the tagged session $\langle 0, (0, i) \rangle$ falls within an output set other than $O_{(1/2)n+1}$.

in the following theorem.

Theorem 2: For a NB- $\log_2(f, N, p)$ network with Banyan stage fan-out and $1 < f \leq N$, a necessary condition for the network to be RNB is:

$$p \geq \begin{cases} \sqrt{N} + 1 & \text{if } n \text{ is even} \\ \sqrt{2N} & \text{if } n \text{ is odd.} \end{cases} \quad (4)$$

Proof: The main idea of our proof is to find the maximum number of connections that conflict with a tagged session and also share a common SE (thus they can not share any plane), then this maximum number plus one (the one for tagged session) will be a lower bound on the number of planes required for the NB- $\log_2(f, N, p)$ network to be RNB.

We first consider the case when $n = \log_2 N$ is even (please refer to Fig. 4). When $f = 2$, the proof is just the generalization of the Example Two. We focus again on a tagged 2-cast session $\langle 0, (0, i) \rangle$ with $i \in O_{(1/2)n+1}$. Then each of these connections from $I_{(1/2)n+1}$ to $\bigcup_{j=1}^{(1/2)n+1} O_j - \{i\}$ will conflict with the tagged session at stage $(1/2)n + 1$, and all these conflicts can block as many as $\min \left(|I_{(1/2)n+1}|, \sum_{j=1}^{(1/2)n+1} |O_j| - 1 \right) = |I_{(1/2)n+1}| = \sqrt{N}$ planes since they all share a common SE at the stage $(1/2)n$. We need one more plane to route the tagged session, so a necessary condition for the network to be RNB becomes $p \geq \sqrt{N} + 1$. In the above analysis, we confine the output i of the tagged 2-cast session $\langle 0, (0, i) \rangle$ within the output set $O_{(1/2)n+1}$. It is easy to verify that if the output i of the tagged session falls within an output set other than $O_{(1/2)n+1}$, the maximum number of connections that conflict with the tagged session and share a common SE will be $\sqrt{N} - 1$, which is not worse than the case when i falls within $O_{(1/2)n+1}$. An example of 16×16 network ($N = 16$ and $n = 2$) is illustrated in the Fig. 5 for the case when i of the tagged 2-cast session $\langle 0, (0, i) \rangle$ falls within an output set $O_{(1/2)n}$ (O_2 there) other than $O_{(1/2)n+1}$, where the maximum number of connections that conflict with the tagged session and share a common SE (dashed one) is $\sum_{j=1}^{(1/2)n} |I_j| = \sqrt{N} - 1 = 3$.

Actually, the conflicting scenario shown in the Fig. 5 is also valid for the cases when i of the tagged 2-cast session

$\langle 0, (0, i) \rangle$ falls within the output sets O_1 or O_4 . When $f \geq 2$, the necessary condition $p \geq \sqrt{N} + 1$ still holds since $f = 2$ is just a special case of $f \geq 2$. In fact, for a tagged f -cast session ($f \geq 2$) that originates from input 0 and includes the connection $[0, 0]$, the maximum number of connections that conflict with the tagged session at stage m will be $\sum_{j=1}^m |I_j| - 1$

for $1 \leq m \leq (1/2)n$ and $\sum_{j=1}^{n-m+1} |O_j| - 1$ for $(1/2)n + 1 \leq m \leq n$. In both cases, the maximum number of connections that conflict with the tagged session and share a common SE will be no more than $\sqrt{N} - 1$. Again, the worst case conflicting with the tagged session happens at stage $(1/2)n + 1$ where \sqrt{N} planes can be blocked simultaneously by the connections from $I_{(1/2)n+1}$.

When $n = \log_2 N$ is odd (please refer to Fig. 1), the proof is similar to that of the case when n is even except that the worst case conflicting with a tagged session now happens at stage $(1/2)(n + 1)$. When $f = 2$, we focus on a tagged 2-cast session $\langle 0, (0, i) \rangle$ when the i falls within an output set other than $\bigcup_{j=1}^{(1/2)(n+1)} O_j$. For the tagged session, the maximum number of connections that conflict with the tagged session and also share a common SE is $\sum_{j=1}^{(1/2)(n+1)} |I_j| = \sum_{j=1}^{(1/2)(n+1)} |O_j| = \sqrt{2N} - 1$, and one such

a worst case scenario can be achieved when all the inputs from set $\bigcup_{j=1}^{(1/2)(n+1)} I_j$ generate unicast requests and they are destined to the output set $\bigcup_{j=1}^{(1/2)(n+1)} O_j$. Adding one more plane for tagged session will result in the necessary condition in (4). It is interesting to notice that above worst case for $f = 2$ is actually the same as that for $f = 1$. In the above analysis, we confine the output i of the tagged 2-cast session $\langle 0, (0, i) \rangle$ within an output set other than $\bigcup_{j=1}^{(1/2)(n+1)} O_j$. This constraint on i is due to the fact that if i falls within the output set $\bigcup_{j=1}^{(1/2)(n+1)} O_j$, the maximum number of connections that conflict with the tagged session and share a common SE will be $\sum_{j=1}^{(1/2)(n+1)} |O_j| - 1 = \sqrt{2N} - 2$,

which is not worse than the case when i falls within an output set other than $\bigcup_{j=1}^{(1/2)(n+1)} O_j$. An example of the worst case conflicting with a tagged session $\langle 0, (0, 8) \rangle$ is illustrated in Fig. 1 for a NB- $\log_2(2, 32, p)$ network ($f = 2$ and $n = 5$), in which all inputs from set $\bigcup_{j=1}^3 I_j$ are destined to the output set $\bigcup_{j=1}^3 O_j$ and all these connections can block as many as 7 planes since they all share the dashed SE. When $f \geq 2$, the necessary condition $p \geq \sqrt{2N}$ still holds since $f = 2$ is just a special case of $f \geq 2$. Another example of the worst case conflicting with a tagged session $\langle 0, (0, 4, 8) \rangle$ is illustrated in Fig. 6 for a NB- $\log_2(3, 32, p)$ network ($f = 3$), in which all inputs from set $\bigcup_{j=1}^3 I_j$ are destined to the output set O_4 and all these connections can block 7 planes since they all share the dashed SE, and this number of blocked planes is just the same as that of the NB- $\log_2(2, 32, p)$ network with $f = 2$. QED.

Based on a similar treatment as that of Theorem 2 and considering only the link-blocking instead of node-blocking in analysis, we can get the following condition for LB- $\log_2(f, N, p)$ networks.

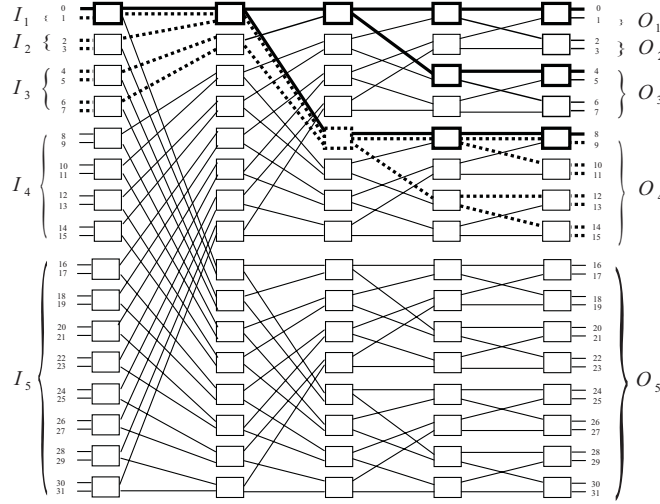


Fig. 6. Illustration of the worst case conflicting for a NB- $\log_2(3, 32, p)$ network with $f = 3$.

Corollary 2: For a LB- $\log_2(f, N, p)$ network with Banyan stage fan-out and $1 < f \leq N$, a necessary condition for the network to be f -cast RNB is:

$$p \geq \begin{cases} \sqrt{N} & \text{if } n \text{ is even} \\ (1/2)\sqrt{2N} + 1 & \text{if } n \text{ is odd.} \end{cases} \quad (5)$$

Remark 4.1: Similar to their full fanout counterparts, it is also a very challenging problem to prove that when the constraint of only Banyan stage fan-out is applied, whether the conditions in Theorem 2 and Corollary 2 are sufficient or not for the general f -cast $\log_2(N, 0, p)$ and $\log_2(N, n, p)$ networks to be RNB, respectively. This remains the future work.

Remark 4.2: We should note that for a $\log_2(f, N, p)$ network with $f = 1$ (unicast), no fan-out is required at any stage of the network, so its RNB condition is always same as its unicast counterpart no matter what the fan-out constraint is. However, the conditions in (4) and (5) indicate that in the case of Banyan stage fan-out and when $f > 1$, a NB- $\log_2(f, N, p)$ network with n being even and a LB- $\log_2(f, N, p)$ network with n being odd require at least one plane more than their unicast counterparts to guarantee RNB.

V. REARRANGEABLE $\log_2(F, N, P)$ NETWORKS WITH INPUT STAGE FAN-OUT

Under the constraint that fan-out capability is only available at the input stage, all the requests of a session will be routed through distinct planes. Due to this strict constraint on fan-out capability, we are able to find the necessary and also sufficient conditions for $\log_2(f, N, p)$ networks to be RNB, as summarized in the following Theorem 3 and Corollary 3.

Theorem 3: A NB- $\log_2(f, N, p)$ network with input stage fan-out is RNB for $1 \leq f \leq N$ if and only if :

$$p = \max \left\{ f \cdot 2^{\lceil (1/2)(n-1-\log_2 f) \rceil}, 2^{\lfloor (1/2)(n+1+\log_2 f) \rfloor} \right\} \quad (6)$$

Proof: We first prove the necessity of this theorem. For a NB- $\log_2(f, N, p)$ network with input stage fan-out, all the requests of a session will be routed independently through

distinct planes. Therefore, we just need to focus on only one of these requests in our analysis and we regard the path of the selected request as the tagged path. Here we focus again on the tagged path between input 0 and output 0. Under the crosstalk-free constraint, we will focus on the tagged SEs and consider only the crosstalk blocking (node blocking) in our analysis.

For a given $f(1 \leq f \leq N)$, we can determine a unique stage j such that all the requests from input set $\bigcup_{k=0}^{j-1} I_k$ can never use up all the outputs in output set $\bigcup_{k=1}^{n-j+2} O_k$ but the requests from input set $\bigcup_{k=0}^j I_k$ may use up all the outputs in output set $\bigcup_{k=0}^{n-j+1} O_k$, where I_0 contains only the input 0. That is, for a given $f(1 \leq f \leq N)$, we can determine a unique stage j such that

$$\sum_{k=0}^{j-1} |I_k| \cdot f - 1 < \sum_{k=1}^{n-j+2} |O_k|$$

and

$$\sum_{k=0}^j |I_k| \cdot f - 1 \geq \sum_{k=1}^{n-j+1} |O_k|$$

where $|I_0| \cdot f - 1$ is just the maximum number of contentions from the same session of the tagged path.¹ The parameter j is illustrated in Fig. 7 for a NB- $\log_2(f, 64, p)$ network with $f = 3$ and $n = 6$, where the j is equal to 3. The dashed requests from the input set $\bigcup_{k=0}^j I_k = \bigcup_{k=0}^3 I_k$ can use up all the dashed outputs in the output set $\bigcup_{k=1}^{n-j+1} O_k = \bigcup_{k=1}^4 O_k$, but the requests from the input set $\bigcup_{k=0}^{j-1} I_k = \bigcup_{k=0}^2 I_k$ can never use up all the outputs in output set $\bigcup_{k=1}^5 O_k$.

The above two inequalities indicate that the f satisfies

$$2^{n-2j+1} \leq f < 2^{n-2j+3}, j = 1, \dots, \left\lceil \frac{n+1}{2} \right\rceil \quad (7)$$

Since the upper bound for f indicates that

$$f < 2^{n-2i+1}, \text{ for } 0 \leq i \leq j-1.$$

Thus, we always have

$$\begin{aligned} \sum_{k=0}^i |I_k| \cdot f - 1 &< 2^i \cdot 2^{n-2i+1} - 1 \\ &= \sum_{k=1}^{n-i+1} |O_k|, \text{ for } 0 \leq i \leq j-1. \end{aligned}$$

The above inequality implies that the node utilization factors of tagged SEs² from stage 0 to stage $j-1$ are determined by the input intersecting sets (Note that the inserting set of stage 0 is just I_0).

¹Under the constraint of input stage fan-out, all the requests of an f -cast session must be routed independently through distinct planes. Therefore, in the blocking analysis of tagged path, we can regard the other connections from the same session of the tagged path as the contentions to the tagged one.

²The node utilization factor is defined in a similar way as link utilization factor [25], in which the utilization factor of a node is just the maximum number of unicast connections that can pass through this node.

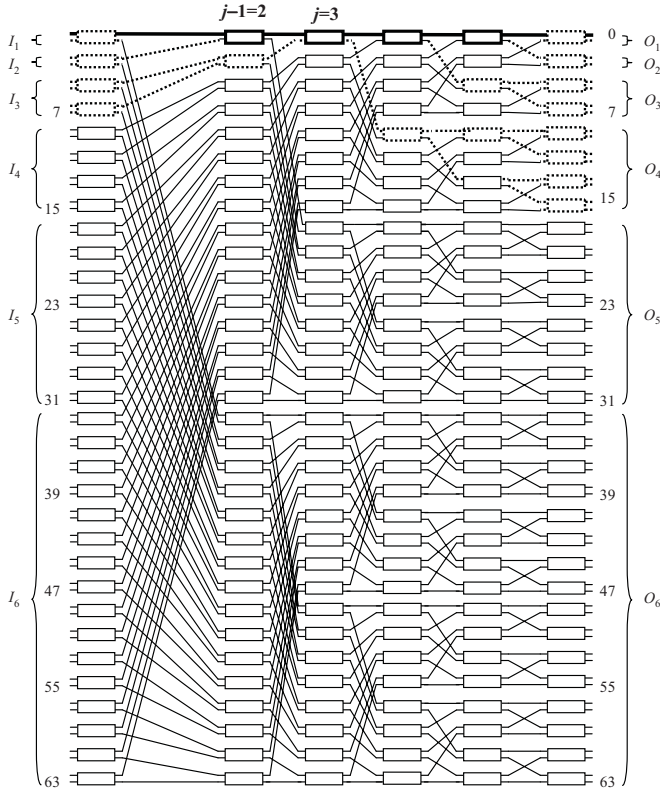


Fig. 7. Illustration of the parameter j for a NB- $\log_2(f, 64, p)$ network with $f = 3$ and $n = 6$.

Similarly, we can see that the lower bound in (7) indicates that

$$\begin{aligned} \sum_{k=0}^l |I_k| \cdot f - 1 &\geq 2^l \cdot 2^{n-2l+1} - 1 \\ &= \sum_{k=1}^{n-l+1} |O_k|, \text{ for } j \leq l \leq n. \end{aligned}$$

Therefore, the node utilization factors of tagged SEs from stage j to stage n are determined by the output intersecting sets.

Notice that a lower bound on the number of planes required for a RNB NB- $\log_2(f, N, p)$ network is just the maximum possible node utilization factor of the tagged path plus one (the one for the tagged connection), so the necessary condition to guarantee the NB-NB- $\log_2(f, N, p)$ network to be RNB under the input fan-out is

$$\begin{aligned} p &\geq \max_{0 \leq i \leq n} \left\{ \min \left(\sum_{k=0}^i |I_k| \cdot f - 1, \sum_{k=1}^{n-i+1} |O_k| \right) \right\} + 1 \\ &= \max_{0 \leq i \leq n} \left\{ \min (2^i \cdot f - 1, 2^{n-i+1} - 1) \right\} + 1 \\ &= \max \{ 2^{j-1} \cdot f - 1, 2^{n-j+1} - 1 \} + 1 \\ &= \max \{ 2^{j-1} \cdot f, 2^{n-j+1} \} \end{aligned} \quad (8)$$

It is notable that the inequality (7) indicates that

$$\frac{n+1-\log_2 f}{2} \leq j < \frac{n+1-\log_2 f}{2} + 1 \quad (9)$$

Since for any $a > 0$, $\lceil a \rceil$ is the only integer that satisfies

$$a \leq \lceil a \rceil < a + 1 \quad (10)$$

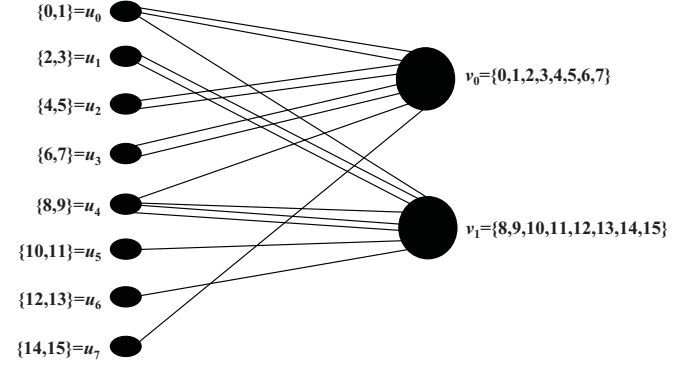


Fig. 8. Bipartite graph for the request frame of Example One, where $f = 2$, $n = 4$ and $j = 2$.

Thus, from (9) we know that

$$j = \left\lceil \frac{n+1-\log_2 f}{2} \right\rceil \quad (11)$$

Note that for any $a > 0$, $\lceil a \rceil - 1 \lceil a - 1 \rceil$, so

$$\begin{aligned} j-1 &= \left\lceil \frac{n+1-\log_2 f}{2} \right\rceil - 1 \\ &= \left\lceil \frac{n-1-\log_2 f}{2} \right\rceil \end{aligned} \quad (12)$$

Inequality (10) indicates that

$$-a-1 < -\lceil a \rceil \leq -a \quad (13)$$

The above inequality (13) implies that

$$\begin{aligned} \frac{n-1+\log_2 f}{2} - 1 &< n+1 - \left\lceil \frac{n+1-\log_2 f}{2} \right\rceil \\ &\leq \frac{n+1+\log_2 f}{2} \end{aligned} \quad (14)$$

Since for any $a > 0$, $\lceil a \rceil$ is the only integer that satisfies $a-1 < \lceil a \rceil \leq a$. Thus, the equation (11) and the inequality (14) indicate that

$$\begin{aligned} n-j+1 &= n+1 - \left\lceil \frac{n+1-\log_2 f}{2} \right\rceil \\ &= \left\lceil \frac{n+1+\log_2 f}{2} \right\rceil \end{aligned} \quad (15)$$

Substituting (12) and (15) into (8) will finish the proof of the necessity of this theorem.

To prove the sufficiency of this theorem, we just need to prove that (or equivalently $\max\{f \cdot 2^{\lceil (n-1-\log_2 f)/2 \rceil}, 2^{\lfloor (n+1+\log_2 f)/2 \rfloor}\}$) number of planes is enough for us to route any request frame without blocking (node-blocking). Here, we explore the edge-coloring property for bipartite multi-graph in our proof.

For a given f and its corresponding j determined by equation (11), we construct an undirected bipartite multi-graph $G = (U \cup V; E)$ for any request frame as follows. Define the vertex set $U = \{u_0, u_1, \dots, u_{2^{n-j+1}-1}\}$ as a partition of input set $\{0, 1, \dots, N-1\}$ and vertex set $V = \{v_0, v_1, \dots, v_{2^j-1}\}$ as a partition of output set $\{0, 1, \dots, N-1\}$, where we get (16) (see top of next page).

We define E as the set of edges between U and V , where a connection originating from u_i and destined for v_l

$$\begin{aligned}
u_i &= \{i \cdot 2^{j-1}, i \cdot 2^{j-1} + 1, \dots, i \cdot 2^{j-1} + 2^{j-1} - 1\}, \\
i &= 0, 1, \dots, 2^{n-j+1} - 1, \\
v_l &= \{l \cdot 2^{n-j+1}, l \cdot 2^{n-j+1} + 1, \dots, l \cdot 2^{n-j+1} + 2^{n-j+1} - 1\}, \\
l &= 0, 1, \dots, 2^{j-1} - 1
\end{aligned} \tag{16}$$

corresponds to an edge in E between nodes u_i and v_l in the bipartite graph. Fig. 8 illustrates the bipartite multi-graph $G = (U \cup V; E)$ for the request frame of Example 1.

From the definition of the bipartite multi-graph $G = (U \cup V; E)$ we know that each vertex in U corresponds to an input set of size 2^{j-1} , so the vertex has degree at most $f \cdot 2^{j-1}$ for the f -cast request frame. Each vertex in V corresponds to an output set of size 2^{n-j+1} , so it has degree at most 2^{n-j+1} . Therefore, the bipartite multi-graph $G = (U \cup V; E)$ defined above has the maximum degree at most $\max\{f \cdot 2^{j-1}, 2^{n-j+1}\}$. By Konig's line coloring theorem [28], the graph G can be properly edge-colored with at most $\max\{f \cdot 2^{j-1}, 2^{n-j+1}\}$ colors such that all vertices in G are incident to edges of different colors. The coloring of G guarantees that the connections assigned with a same color will be node disjoint in both vertex sets U and V . Based on the definition of u_l and v_l in (16) and the topology property of Banyan networks, node disjoint property in U and node disjoint property in V guarantee that all the connections with the same color will be SE-disjoint (thus nonblocking) from stage 0 to stage $j-1$ and from stage j to stage n , respectively. Therefore, all the connections with the same color can be implemented without blocking in a common plane of NB- $\log_2(f, N, p)$ network. This finishes the proof of sufficiency of this theorem. Q.E.D.

Based on a similar proof as that of the theorem 3 and consider only the link-blocking instead of node-blocking in analysis, we can get the following necessary and also sufficient condition for LB- $\log_2(f, N, p)$ networks.

Corollary 3: A LB- $\log_2(f, N, p)$ network with input stage fan-out is f -cast RNB for $1 \leq f \leq N$ if and only if :

$$p = \max \left\{ f \cdot 2^{\lceil (1/2)(n-2-\log_2 f) \rceil}, 2^{\lfloor (1/2)(n+\log_2 f) \rfloor} \right\}$$

VI. COMPARISON AND DISCUSSION

It is worth noticing that for the $\log_2(f, N, p)$ networks with full fan-out or with Banyan stage fan-out we have available in our hands only the necessary conditions for these networks to be RNB, so we can not provide a complete and fair comparison among the nonblocking conditions of these networks, because their actual necessary and also sufficient conditions may require more planes than the necessary conditions we developed. Therefore, we provide here only the comparison among the $\log_2(f, N, p)$ networks with input stage fan-out based on their necessary and also sufficient conditions obtained.

For the LB- $\log_2(f, N, p)$ and NB- $\log_2(f, N, p)$ networks with only the input stage fan-out, Table I and Table II show the number of planes (p) required for them to be RNB with the variations of network size (N) and parameter f .

The two tables all indicate that the number of planes required for RNB in general increases with the increase of

TABLE I
NUMBER OF PLANES (P) FOR RNB LB- $\log_2(f, N, P)$ NETWORKS WITH INPUT STAGE FAN-OUT

$f \backslash N$	4	8	16	32	64	128	256	512	1024
1	2	2	4	4	8	8	16	16	32
2	2	4	4	8	8	16	16	32	32
3	3	4	6	8	12	16	24	32	48
4		4	8	8	16	16	32	32	64
5		5	8	10	16	20	32	40	64
6		6	8	12	16	24	32	48	64
7		7	8	14	16	28	32	56	64
8			8	16	16	32	32	64	64
9			9	16	18	32	36	64	72
10			10	16	20	32	40	64	80
11			11	16	22	32	44	64	88
12			12	16	24	32	48	64	96
13			13	16	26	32	52	64	104
14			14	16	28	32	56	64	112
15			15	16	30	32	60	64	120
16				16	32	32	64	64	128
17				17	32	34	64	68	128
18				18	32	36	64	72	128
19				19	32	38	64	76	128
20				20	32	40	64	80	128
21				21	32	42	64	84	128
22				22	32	44	64	88	128
23				23	32	46	64	92	128
24				24	32	48	64	96	128
25				25	32	50	64	100	128
26				26	32	52	64	104	128
27				27	32	54	64	108	128
28				28	32	56	64	112	128
29				29	32	58	64	116	128
30				30	32	60	64	120	128

f for both LB- $\log_2(f, N, p)$ and NB- $\log_2(f, N, p)$ networks, and a RNB NB- $\log_2(f, N, p)$ network always requires more planes than its LB- $\log_2(f, N, p)$ counterpart due its stricter node-blocking constraint. A careful observation of both tables further indicates that for some ranges of f , the number of planes p required by a RNB $\log_2(f, N, p)$ network increases monotonously with the increase of f in these ranges, while the parameter p keeps unchanged with the increase of f when f falls within other ranges. These two kinds of ranges of f appear alternatively. For example in LB- $\log_2(f, 256, p)$ networks, the p increases monotonously with f when the latter is in the range $8 \leq f \leq 16$, while the p is always equal to 64 for the range $16 \leq f \leq 30$ of f . Similar behaviors can also be observed in the NB- $\log_2(f, 256, p)$ networks.

We can also find from both tables that providing the fan-out

TABLE II
NUMBER OF PLANES (P) FOR RNB NB-LOG₂(F, N, P) NETWORKS
WITH INPUT STAGE FAN-OUT

$f \backslash N$	4	8	16	32	64	128	256	512	1024
1	2	4	4	8	8	16	16	32	32
2	4	4	8	8	16	16	32	32	64
3	4	6	8	12	16	24	32	48	64
4		8	8	16	16	32	32	64	64
5		8	10	16	20	32	40	64	80
6		8	12	16	24	32	48	64	96
7		8	14	16	28	32	56	64	112
8			16	16	32	32	64	64	128
9			16	18	32	36	64	72	128
10			16	20	32	40	64	80	128
11			16	22	32	44	64	88	128
12			16	24	32	48	64	96	128
13			16	26	32	52	64	104	128
14			16	28	32	56	64	112	128
15			16	30	32	60	64	120	128
16				32	32	64	64	128	128
17				32	34	64	68	128	136
18				32	36	64	72	128	144
19				32	38	64	76	128	152
20				32	40	64	80	128	160
21				32	42	64	84	128	168
22				32	44	64	88	128	176
23				32	46	64	92	128	184
24				32	48	64	96	128	192
25				32	50	64	100	128	200
26				32	52	64	104	128	208
27				32	54	64	108	128	216
28				32	56	64	112	128	224
29				32	58	64	116	128	232
30				32	60	64	120	128	240

function only in the input stage of a $\log_2(f, N, p)$ network can be a reasonable choice when f is small (e.g., less than 5), since we don't need too many number of planes to guarantee RNB and the complex SE architecture with fan-out capability is not required in the central stage. Also, for a $\log_2(f, N, p)$ with small number of planes, the passive splitters and combiners can be adopted for the input and output stages without increasing the network depth and introducing a heavy splitting (combining) loss. When f is relatively larger, however, a $\log_2(f, N, p)$ network (in particular the NB- $\log_2(f, N, p)$ one) with only the input stage fan-out requires a large number of planes and thus a high overall hardware cost to guarantee the RNB property. In addition, for a $\log_2(f, N, p)$ network with large number of planes, the multistage active splitting at its input stage will be required to achieve a low splitting loss, so the overall network depth will be increased. Thus, for the $\log_2(f, N, p)$ networks with a large value of f , we may need to provide the fan-out capability also in the central Banyan stage to avoid requiring a prohibitively huge number of planes and also a large network depth to guarantee the RNB property. This remains to be explored further.

The current switch designs for supporting one-to-many connections mainly focus on the multicast switches without

any constraint on the maximum fan-out of each connection request, see, for example [5,6,8,10,15], and it is notable that the design of an $N \times N$ multicast switch just corresponds to a special case of its f -cast design (i.e. when $f = N$). The available results in [18] on SNB f -cast $\log_2(f, N, p)$ with full fan-out and our initial results in this paper on RNB f -cast $\log_2(f, N, p)$ with input stage fan-out indicate that by restricting the maximum fan-out of each request to a specified small value f in some switch designs, the corresponding f -cast switch designs may significantly reduce the overall hardware cost than their multicast counterpart. A carefully observation of communication network traffics indicates that almost all the multicast applications are restricted to a small group of users and the unconstrained multicast applications are really occasional, so allocating a very high multicast capability to each network switch and spending a huge amount of extra hardware in order to guarantee the multicast nonblocking property is very cost-inefficient for most cases. Also, from the viewpoints of resource fairness and network security (e.g., the efficient control of virus and worms dissemination), we have many good reasons to impose a restriction on the maximum fan-out of each request. Actually, through proper network routing control (e.g. by constructing a multicast tree), the occasional unconstrained can be supported by multiple limited fan-out switches (routers). Therefore, we envision that the study on the general design of f -cast switches, which actually covers the unicast design ($f = 1$) or the multicast design ($f = N$) as two special cases, will offer network designers much more flexibilities and may result in a significant hardware saving for the design of future multicast-intensive networks.

VII. CONCLUSIONS AND FUTURE WORKS

We have studied both the f -cast multi- $\log_2 N$ networks with link-blocking constraint and f -cast multi- $\log_2 N$ networks with node-blocking constraint. We have determined the necessary conditions for these to be RNB for the cases when fan-out capability is available at both the input and Banyan stages or when fan-out capability is available only at Banyan stage, and we have succeeded in obtaining the necessary and also sufficient conditions for the f -cast multi- $\log_2 N$ networks to be RNB for the case when fan-out capability is only available at the input stage. It is surprising that the necessary conditions for the first two fan-out cases are independent of parameter f when $f > 1$, and if $f > 1$ the conditions for second case require at most one Banyan plane more than their unicast counterparts depending on both the blocking constraint (node-blocking or link blocking) and network depth $\log_2 N$ (even or odd). Our future work is to determine the sufficient conditions for the first two fan-out cases either.

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