Joint Design of Network Coding and Transmission Rate Selection for Multihop Wireless Networks

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***Abstract*—Network coding has the potential to greatly improve the throughput of wireless networks. In the current proposals for wireless network coding, network nodes transmit packets at a fixed transmission rate. It is notable, however, that by dynamically selecting the rate, we can effectively improve the node transmission efficiency. In this paper, we study the application of a rate-adaptive transmission mechanism in network-coding-based multihop wireless networks. In such networks, whether a coding solution is satisfactory or not depends not only on the number of involved native packets but on the packet loss probabilities of its intended next hops and its transmission time as well, both of which depend on the transmission rate. Therefore, we aim to jointly design the coding operation and rate selection to maximize the transmission efficiency. Specifically, we first describe and mathematically formulate the optimal packet coding and rate-selection problem. Then, we prove the NP-completeness of this optimization problem. Finally, we propose an efficient algorithm for finding good combinations of the coding solution and the transmission rate. Simulation results demonstrate that compared with the rate-fixed transmission, the rate-adaptive transmission based on our algorithm can significantly improve the node transmission efficiency.**

***Index Terms*—Multihop wireless networks, network coding, rate-adaptive transmission, transmission efficiency.**

I. Introduction

MULTIHOP wireless networks have been an active area of research for many years. Promising applications of such type of networks include wireless sensor networks, wireless mesh networks, etc. One of the most significant problems of multihop wireless networks is that their current implementations suffer from a severe throughput limitation and do not scale well as the number of network nodes increases [1].

The network coding technique is one of a few fundamental approaches that can essentially improve wireless throughput. This technique was originally proposed to save bandwidth and increase the throughput of multicast in wired networks [2], and it was shown later that it can also offer benefits for other connection cases in both wired and wireless networks [3]–[12]. The basic idea of network coding in wireless networks is quite simple and can be illustrated using the scenario in Fig. 1 (from [3]), where node A wants to send packet P_1 to node B, and node B wants to send packet P_2 to node A with the help of intermediate node R. Suppose that node R has received P_1 and P_2. In the traditional transmission way, node R separately transmits P_1 and P_2. However, node R can XOR P_1 and P_2 together and broadcast $P_1 \oplus P_2$. Upon receiving $P_1 \oplus P_2$, node A can decode P_2 by $P_2 = P_1 \oplus (P_1 \oplus P_2)$. Similarly, node B can decode P_1 by $P_1 = P_2 \oplus (P_1 \oplus P_2)$. Therefore, with the network coding function, node R can forward two packets in a single packet transmission, and its transmission efficiency is improved by 100% when P_1 and P_2 are the same size.

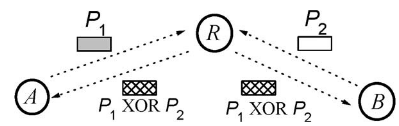

Fig. 1. Simple scenario of wirelesses network coding.

Following the study of the aforementioned basic scenario, recently, Katti *et al.* has proposed the first practical network-coding-based packet forwarding architecture (called COPE) to essentially improve the network throughput of multihop wireless networks [4]. In COPE, each node opportunistically overhears those packets transmitted by its neighbors, which are not addressed to itself, and notices what packets the neighbors currently possess (by adding piggyback reception reports on the data packets the node transmits). Each node can intelligently XOR multiple packets destined to different next hops such that multiple packets can be forwarded in a single transmission, resulting in a significant improvement in node transmission efficiency. Results obtained from the first testbed deployment of wireless network coding showed that COPE can substantially improve the throughput of multihop wireless networks.

In the current proposals (including COPE) for wireless network coding, network nodes transmit packets at a fixed transmission rate. It is notable, however, that, by dynamically selecting the appropriate rate, we can improve the node transmission efficiency [13], [14]. Let us consider a simple example. Suppose that currently the packet loss probability from node R to A is 0.90, 0.70, 0.66, and 0.30 when the selected transmission rate is 1, 2, 5.5, and 11 Mb/s, respectively. Clearly, the expected time required to successfully deliver a packet with size S (in bits) to A is $S/0.9$, $S/1.4$, $S/3.63$, and $S/3.3$ μs when using the rate of 1, 2, 5.5, and 11 Mb/s, respectively.

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Thus, the rate of 5.5 Mb/s is the best solution for the current packet transmission. However, as the packet loss probabilities at different rates change over time due to some factors like node mobility, the best rate also changes over time. Accordingly, by dynamically selecting the appropriate rate, we can effectively reduce the transmission time and improve the node transmission efficiency. Similar to the noncoding packet transmission, by dynamically selecting the transmission rate in the coding-based transmission, we can also improve the node transmission efficiency. In [15], for the transmission of simple coded packets involving only two native packets, Ni *et al.* have preliminarily investigated the gain achieved by dynamically selecting the rate. In their investigation, all packets for coding have the same size. In such a case, the determination of the coding solution is trivial,[1] and the selection of the optimal rate is quite easy.

In this paper, we study the practical application of a rate-adaptive transmission mechanism in network-coding-based multihop wireless networks. In the coding-based transmission scheme, two new features make the rate selection much different from that in the noncoding transmission.

1) In the noncoding transmission, the metric for measuring the efficiency of transmitting a native packet at one specific rate (i.e., for rate selection) is quite simple, as previously introduced. In the coding-based transmission, however, each encoded packet involves multiple native packets, and thus, the metric applied in the noncoding transmission cannot be applied to evaluate the efficiency of transmitting an encoded packet at one specific rate. Since different native packets of one encoded packet have different sizes and different packet loss probabilities (they are destined to different next hops), a reasonable metric for rate selection in the coding-based case should take into account the effects of all involved packets' sizes and the packet loss probabilities of their intended next hops, as well as the transmission time.
2) More importantly, whether a coding solution is satisfactory depends not only on the number of involved native packets but also on the packet loss probabilities at its intended next hops and its transmission time, both of which depend on the transmission rate. It may happen that although an encoded packet involves many packets, packet loss probabilities at its intended next hops are high, no matter which rate is used. Therefore, the determination of the coding solution should be coupled with the selection of the transmission rate such that we can find good combinations of the coding solution and the transmission rate.

Based on the aforementioned two features, in this paper, we first define a metric for measuring the efficiency of transmitting an encoded packet at one specific rate. We then study the joint design of coding operation and rate selection to maximize the node transmission efficiency. The main contributions of this paper are summarized as follows.

1) We mathematically formulate the optimal packet-coding and rate-selection (PC-RS) problem as an integer programming problem, which is NP-complete. The mathematical formulation can help us assess the essence and understand the hardness of this problem well.
2) An efficient heuristic algorithm is proposed to jointly find good combinations of the coding solution and the transmission rate.
3) We demonstrate that compared to the rate-fixed transmission, the rate-adaptive transmission based on our algorithm can significantly improve the node transmission efficiency.

The rest of this paper is organized as follows. Section II describes and mathematically formulates the optimal PC-RS problem. Section III presents a joint PC-RS algorithm. Simulation results are presented in Section IV. Finally, Section V concludes this paper.

II. Optimal Packet-Coding–Rate-Selection Problem

In this section, we first define a metric, i.e., the expected transmission efficiency (ETE), that measures the efficiency of transmitting an encoded packet at one specific rate. We then describe and formulate the optimal PC-RS problem.

A. System Model

Here, we consider the application of network coding in multihop wireless networks. The network nodes are set in promiscuous mode, which helps them snoop on all communications over the wireless medium and store the overheard packets. Each node can know the (overheard and routed) packets each neighbor possesses such that it can make network coding operations based on this information. This can be achieved by using *reception reports*, as introduced in [4]. In addition, it is assumed that each node knows the packet loss probabilities of links from itself to its neighbors, which can be estimated by periodically broadcasting particular control packets and dividing the number of received packets at one neighbor to the number of sent packets [21].

B. ETE

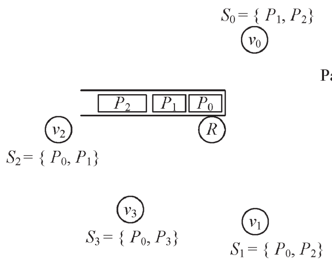
To be able to explicitly measure the "goodness" of transmitting an encoded packet at a specific transmission rate, it is necessary for us to adopt a reasonable metric that should take into account the effects of some important parameters such as each involved packet's size, packet-delivery ratios, and transmission time. Currently, there is no such metric available. Here, we introduce a metric for this purpose.

Definition: For a coding solution $P_0 \oplus \cdots \oplus P_L \ (L \geq 0)$ whose designated receiver in the MAC layer is the intended next hop of P_0, define the ETE γ when using transmission rate r as follows:

$$\gamma = \theta_0 \cdot \frac{l_0 + \sum_{k=1}^{L} \left[1 - (1-\theta_k)^{1/\theta_0}\right] l_k}{\max_{0 \leq k \leq L} l_k/r + T_c} \tag{1}$$

where l_k is the payload size of packet P_k in bits, θ_k is the probability of the intended next hop of P_k successfully receiving

[1]The node only needs to encode the head packet of one flow and the head packet of the other flow together.



Packet	Size (bytes)	Nexthop
P_0	200	v_0
P_1	100	v_1
P_2	1500	v_2
P_3	200	v_1

Rate (Mbps)	θ_0	θ_1	θ_2	θ_3
1	0.95	0.70	0.50	0.90
2	0.90	0.55	0.36	0.85
5.5	0.85	0.40	0.10	0.80
11	0.40	0.20	0.08	0.40

Fig. 2. Example adopted to illustrate the necessity of jointly design of coding operation and rate selection.

the encoded packet when using transmission rate r, and T_c is the (constant) total time consumed in each transmission by the physical-layer convergence procedure header, short interframe space, acknowledgment, etc.

Let us examine what this metric reflects. First, the expected number of transmissions of this encoded packet is $1/\theta_0$. During these $1/\theta_0$ transmissions, it is expected that totally $l_0 + \sum_{k=1}^{L}[1-(1-\theta_k)^{1/\theta_0}]l_k$ bits can successfully be delivered to the next hops. In addition, the payload size of the encoded packet $P_0 \oplus \cdots \oplus P_L$ is approximately equal to that of the largest packet[2] (i.e., $\max_{0 \le i \le N} l_i$), and consequently, each transmission consumes time $\max_{0 \le k \le L} l_k/r + T_c$. Thus, the ratio of $l_0 + \sum_{k=1}^{L}[1-(1-\theta_k)^{1/\theta_0}]l_k$ to $(\max_{0 \le k \le L} l_k/r + T_c)/\theta_0$ (i.e., ETE) accurately reflects the *expected number of delivered bits per second* during the whole transmission (including retransmission) period of this encoded packet. The larger the ETE, the better the current packet transmission.

Having the efficiency metric ETE, once a node obtains one transmission chance, it can first find a coding solution that involves as many native packets as possible and then select the rate that provides the maximal ETE. However, such a coding solution determination way, which is independent from the rate selection, cannot guarantee the quality of the obtained coding solution. We present one example in Fig. 2 to clearly illustrate this insight. The next hop of each packet at node R, which is the set S_i of packets that the next hop v_i already possesses, and the packet delivery ratio θ are shown in this figure. Let us suppose that $T_c = 1.232 \times 10^{-3}$ s, which is obtained from the standard setting of parameters like distributed interframe space. In this example, node R can first encode P_0, P_1, and P_2 together. However, as the packet loss probabilities at P_1 and P_2's intended receivers are high no matter which rate is selected, the maximal ETE that node R can achieve is only 8.42×10^5 (when using the rate of 2 Mb/s), which is even lower than that achieved by transmitting P_0 alone at the rate of 5.5 Mb/s (8.93×10^5). From this example, we can know that whether a coding solution is satisfactory depends not only on the number of involved native packets but on the packet loss probabilities of its intended next hops and its transmission time as well, both of which are related to the specific transmission rate selected. It may happen that although an encoded packet involves many packets, packet loss probabilities at its intended next hops are high, no matter which rate is used. Motivated by this observation, we aim to jointly design the coding operation and rate selection to maximize the coding gain, i.e., find good combinations of the coding solution and the transmission rate to achieve a large ETE. In this example, if we encode P_0 and P_1 together and select the rate of 5.5Mb/s, we can achieve the maximal ETE of 1.09×10^6, which is much larger than that achieved when transmitting P_0 alone.

[2]In each encoded packet's header, several symbols are used to record the number of native packets that are XORed together, IDs of native packets, etc.

C. Formulation of the Optimal PC-RS Problem

We say that an encoded packet (i.e., a coding solution) is *feasible* if it can be decoded by each of its intended next hops. Then, the optimal PC-RS problem can be formally described as follows: Given the packets destined to each next hop, the packets that each next hop already possesses, and each next hop's packet loss probabilities at different transmission rates, find the feasible coding solution and its corresponding transmission rate that jointly provide the maximal short-term transmission efficiency γ.

To mathematically formulate the PC-RS problem, we first introduce some notations. Notations employed in the optimal packet coding problem are summarized in Table I. N_r is the number of available transmission rates. v_i represents the downstream neighbor i, and M is the number of downstream neighbors, except for v_0. Let P_0 be the packet at the head of output queue, which needs to be transmitted in the current transmission. Without loss of generality, we suppose that the next hop of packet P_0 is downstream neighbor v_0. We call R_i in Table I the downstream neighbor v_i's *requirement set*. Since the current COPE architecture only supports unicast traffic (i.e., each flow only goes to one neighbor), we have $R_i \cap R_j = \emptyset$, $\forall 0 \le i \ne j \le M$. We call S_i in Table I downstream neighbor v_i's *storage set*.

Based on the notations in Table I, the PC-RS problem can be formally formulated as shown in Fig. 3. Obviously, the aforementioned PC-RS problem is an integer programming problem. In this formulation, we can see that if parameter $x_i = 1$, then packet P_i will be encoded with P_0; if parameter $y_i = 1$, then rate r_i will be selected. Constraint (3) enforces that exactly one transmission rate will be selected, whose corresponding y_i equals 1.

Constraint (4) enforces that next hop v_0 can retrieve packet P_0 from the encoded packet. In this constraint, X is the coding vector. The first element of X is set to 1 because P_0 (head packet of output buffer) is definitely one of those packets to be encoded. Each possessed native packet at v_0 can be regarded as one corresponding unit vector. For example, P_1 is regarded as unit vector $[0, 1, 0, \ldots, 0]$ (i.e., vector $\mathbf{e}_2$). Then, the condition that v_0 can decode the encoded packet with its possessed packets is reduced to the condition that v_0 can obtain unit vector $[1, 0, \ldots, 0]$ by XORing X with its possessed unit vectors, which is reflected in (4). Clearly, this constraint enforces that all other native packets participating in the coding, except for P_0, must belong to $S_0 = \{P_{0,1}^s, \ldots, P_{0,m_0}^s\}$.

TABLE I
NOTATIONS EMPLOYED TO DESCRIBE THE OPTIMAL PACKET-CODING PROBLEM

Notation	Meaning
N_r	number of available transmission rates.
v_i	downstream neighbor i.
P_0	packet which needs to be delivered in the current transmission. Its nexthop is v_0.
R_0	packet set $\{P_0\}$.
M	number of downstream neighbors except v_0.
n_i	number of candidate packets destined to downstream neighbor v_i. $n_i \geq 1$.
R_i	set $\{P^r_{i,1}, \cdots, P^r_{i,n_i}\}$ of candidate packets destined to downstream neighbor v_i. $(1 \leq i \leq M)$
K	total number of candidate packets for coding with P_0 (i.e., $K = \sum_{i=1}^{M} n_i$).
R	candidate packet set $\bigcup_{i=0}^{M} R_i = \{P_0, P_1, \cdots, P_K\}$.
$p_{k,i}$	probability that a packet transmitted by the current node at rate r_i can be successfully received by v_k.
l_i	size of packet P_i.
S_i	set $\{P^s_{i,1}, \cdots, P^s_{i,m_i}\}$ of packets that v_i possesses ($S_i \subseteq R \backslash R_i$). $(0 \leq i \leq M)$
m_i	number of packets in S_i.
$g(P^r_{i,j}) = n$	the mapping function from packet $P^r_{i,j}$ to its ID n in R.
$\mathbf{e}_i$	the i-th unit vector of dimension $K+1$. $(1 \leq i \leq K+1)$

Given: N_r, R_i, l_i, S_i and $p_{i,j}$.
Encoded packet: $P = P_0 \oplus x_1 P_1 \oplus \cdots \oplus x_K P_K$
Maximize:

$$\gamma = \sum_{i=1}^{N_r} y_i \gamma_i = \sum_{i=1}^{N_r} y_i \frac{l_0 + (x_1 l_1 + \cdots + x_{n_1} l_{n_1})[1-(1-p_{1,i})]^{1/p_{0,i}} + \cdots + (x_m l_m + \cdots + x_K l_K)[1-(1-p_{M,i})]^{1/p_{0,i}}}{\max\{l_0, x_1 l_1, \cdots, x_K l_K\}/r_i + T_c}. \quad (2)$$

where $m = 1 + \sum_{i=1}^{M-1} n_i$.
Over variables:
$x_i \in \{0,1\} : 1 \leq i \leq K,\ \ y_i \in \{0,1\} : 1 \leq i \leq N_r$
$k^t_i \in \{0,1\} : 1 \leq t \leq M,\ 1 \leq i \leq n_t$
$k^t_{i,j} \in \{0,1\} : 0 \leq t \leq M,\ 1 \leq i \leq n_t,\ 1 \leq j \leq m_t\ (n_0 = 1)$
Subject to:

$$\sum_{i=1}^{N_r} y_i = 1. \quad (3)$$

0) Constraints that ensure nexthop v_0 can decode its intended packet P_0:

$$X \oplus k^0_{1,1} \mathbf{e}_{g(P^s_{0,1})+1} \oplus \cdots \oplus k^0_{1,m_0} \mathbf{e}_{g(P^s_{0,m_0})+1} = \mathbf{e}_1 \quad (4)$$

where $X = [1, x_1, \cdots, x_K]$.
1) Constraints that ensure nexthop v_1 can decode its intended packet (if it exists):

$$k^1_i X \oplus k^1_{i,1} \mathbf{e}_{g(P^s_{1,1})+1} \oplus \cdots \oplus k^1_{i,m_1} \mathbf{e}_{g(P^s_{1,m_1})+1} = x_{g(P^r_{1,i})} \mathbf{e}_{g(P^r_{1,i})+1},\ \forall 1 \leq i \leq n_1, \quad (5)$$

2) $\cdots$

M) Constraints that ensure nexthop v_M can decode its intended packet (if it exists):

$$k^M_i X \oplus k^M_{i,1} \mathbf{e}_{g(P^s_{M,1})+1} \oplus \cdots \oplus k^M_{i,m_M} \mathbf{e}_{g(P^s_{M,m_M})+1} = x_{g(P^r_{M,i})} \mathbf{e}_{g(P^r_{M,i})+1},\ \forall 1 \leq i \leq n_M. \quad (6)$$

Fig. 3. Mathematical formulation of the PC-RS problem.

The constraint set (5) in Fig. 3 enforces that next hop v_1 can retrieve its packet from the encoded packet if one of $R_1 = \{P^r_{1,1}, \ldots, P^r_{1,n_1}\}$ is encoded with P_0. For v_1, suppose that $P^r_{1,1}$ is encoded with P_0, i.e., $x_{g(P^r_{1,1})}$ is set to 1. Then, the first constraint (i.e., the one corresponding to $i = 1$) in constraint set (5) enforces that v_1 can retrieve $P^r_{1,1}$ with its possessed unit vectors. It is easy to know that if two or more packets among R_1 are encoded with P_0, v_1 cannot retrieve any packet from the encoded packet by XORing its possessed unit vectors with X (i.e., the constraints in (5) are not satisfied).

Similar constraints are also applied to other downstream neighbors. Finally, in the objective function [see (2) in Fig. 3], the ETE achieved when using the selected coding solution and transmission rate is evaluated.

The following theorem demonstrates that the PC-RS problem is actually NP-complete.

Theorem 1: The PC-RS problem is NP-complete.

Proof: To prove that the general PC-RS problem is NP-complete, it suffices to prove that its following subproblem (called the sub-PC-RS problem) is NP-hard: Given that rate r_1

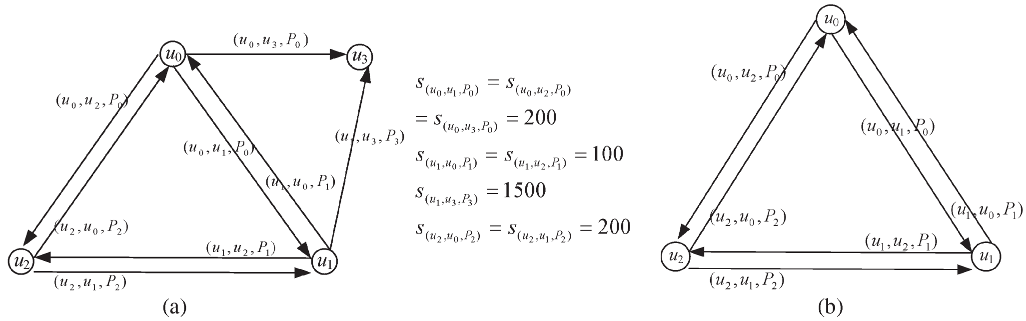


Fig. 4. Joint coding and rate-selection algorithm. (a) Coding graph corresponding to the coding problem in Fig. 2. (b) Example of a feasible coding graph.

is selected (i.e., $y_1 = 1$), find the coding solution having the maximal ETE. It is easy to know that the sub-PC-RS problem belongs to NP. Therefore, it is enough to show a polynomial-time reduction from the maximum clique (MC) problem described below (one of the typical NP-complete problems [16]) to the sub-PC-RS problem.

Instance: An undirected graph $G = (V, E)$, and a positive integer $k \leq |V|$.
Question: Is there a set of k mutually adjacent nodes?

Here is the reduction. Given an arbitrary instance $G = (V, E)$ of the MC problem, where $V = \{u_1, u_2, \ldots, u_n\}$ and E is the set of edges, we construct a corresponding instance of the sub-PC-RS problem as follows. Let the number of downstream neighbors in the sub-PC-RS problem be $n + 1$, and denote them by $v_0, v_1, \ldots, v_n$. Let $R_0 = \{P_0\}$ and $S_0 = \{P_1, P_2, \ldots, P_n\}$, where P_0 is the packet at the head of the output queue. For each $i \in \{1, 2, \ldots, n\}$, let $R_i = \{P_i\}$ and $S_i = \{P_0\} \cup \{P_j : (i, j) \in E\}$. All packets have the same size. Each packet delivery ratio $p_{k,i}$ is approximately equal to 1.

Based on the aforementioned construction of the sub-PC-RS problem, we can know that the answer to the instance of the MC problem is "YES" if and only if there is a feasible packet coding of $k + 1$ packets for the corresponding instance of the sub-PC-RS problem. ■

III. Joint Packet-Coding–Rate-Selection Algorithm

No polynomial-time algorithms are available to obtain the optimal solution of an NP-complete optimization problem. It is necessary for us to develop efficient heuristic algorithms to deal with it. In this section, we show that, due to some unique properties of the PC-RS problem, it is possible for us to design an efficient algorithm for finding good combinations of the coding solution and the transmission rate.

The PC-RS problem has the following important properties.

P1: For an arbitrary candidate packet, suppose that it is destined to the next hop v_i. Then, it definitely cannot be encoded with P_0 if at least one of the following two conditions is not satisfied: 1) v_0 possesses this packet, and 2) v_i possesses P_0.

P2: Among all possible coding solutions, only the solutions that encode native packets destined to different next hops need to be considered, because if two or more packets destined to the same next hop are encoded together, their next hop cannot decode the encoded packet.

P3: In most cases, the number of native packets we can encode together is not large. We can still achieve good performance by encoding at most four native packets together in all cases, as indicated in [4].

Based on the aforementioned important properties of the PC-RS problem, we propose a joint PC-RS algorithm. The main idea is to first construct a directed graph corresponding to the original PC-RS problem, such that the search of coding solutions is reduced to the search of special subgraphs in this graph. Then, we apply the aforementioned properties P1 and P2 to quickly remove arcs and nodes in the graph such that the search space of coding solutions can quickly be shrunk. The simplification of the graph and searching for subgraphs will be repeated at most three times by applying the aforementioned property P3.

Several main procedures of our coding algorithm are described as follows.

Procedure 1 (Graph Construction)

Based on input information like R_i and S_i, construct a directed graph $\mathcal{G}(\mathcal{N}, \mathcal{A})$ (referred to as a *coding graph* henceforth). The node set $\mathcal{N}$ of $\mathcal{G}$ is defined as $\mathcal{N} = \{u_0, u_1, \ldots, u_M\}$, where node u_i corresponds to downstream neighbor v_i in the PC-RS problem and has N_r different weights: $z^1_{(u_i)} = p_{i,1}, \ldots, z^{N_r}_{(u_i)} = p_{i,N_r}$. The arc set $\mathcal{A}$ of $\mathcal{G}$ is defined as follows: for each packet $P^r_{i,k}$ in each R_i, for any $u_j (j \in \{0, 1, \ldots, M\} \setminus \{i\})$ satisfying $P^r_{i,k} \in S_j$, there is a corresponding directed arc $(u_i, u_j, P^r_{i,k})$. Each arc $(u_i, u_j, P^r_{i,k})$ has a weight $s_{(u_i, u_j, P^r_{i,k})}$ equaling the packet size $l_{g(P^r_{i,k})}$.

Let us take the coding problem in Fig. 2 as an example to briefly show how to construct the coding graph. There are four downstream neighbors. Thus, we have four nodes u_0, u_1, u_2, and u_3. The next hop of packet P_0 is v_0. As v_1, v_2, and v_3 already possess P_0, we build arcs (u_0, u_1, P_0), (u_0, u_2, P_0), and (u_0, u_3, P_0). The weights of these three arcs are equal to the size of P_0, i.e., 200. Similarly, for other packets, we build corresponding arcs. Finally, the obtained coding graph is shown in Fig. 4(a).

For a subgraph of $\mathcal{G}$, we call it a *feasible* coding subgraph if we have the following:

1) It contains u_0.
2) For any two different nodes u_i and u_j in it, these is exactly one arc from u_i to u_j, as well as one arc from u_j to u_i.
3) All arcs departing from the same node have the same label P.

Note that a feasible coding subgraph in $\mathcal{G}$ corresponds to a feasible coding solution. Fig. 4(b) shows a simple feasible coding subgraph, whose corresponding feasible coding solution is $P_0 \oplus P_1 \oplus P_2$. We further call a feasible coding subgraph with k nodes a k-node feasible coding subgraph.

Procedure 2 (Property-P1-Based Graph Simplification)

In this step, we apply property P1 to quickly remove some arcs and nodes in the coding graph $\mathcal{G}$. First, remove any node u_i $(1 \leq i \leq M)$ and all its adjacent arcs if there is no arc from u_0 to u_i. Second, remove any u_i and all its adjacent arcs if there is no arc from it to u_0. Third, for any packet P_i, if there does not exist such an arc that arrives at u_0 with label P_i, remove all arcs with label P_i.

In Fig. 4(a), as there does not exist an arc from u_3 to u_0, we remove u_3 and its adjacent arcs (u_0, u_3, P_0) and (u_1, u_3, P_3).

Procedure 3 (Graph Simplification Before Searching for k-Node Feasible Subgraphs)

Here, the properties of k-node feasible subgraphs are applied to further simplify the coding graph $\mathcal{G}$.

1) Remove each arc (u_i, u_j, P) with capacity $c_{(u_i,u_j,P)} < k-1$, where $c_{(u_i,u_j,P)}$ is the total number of arcs that leave from u_i and have the same label P.
2) Remove a node and all its adjacent arcs if its in-degree is less than $k-1$. Here, the *in-degree* of u_i is the number of nodes from which there exist one or more arcs to node u_i.
3) Remove a node and all its adjacent arcs if there is no arc from it to u_0.

These three steps are repeated until 1) $\mathcal{G}$ does not contain u_0 or has less than k nodes, in which case, it then returns FALSE; or 2) nodes and arcs in $\mathcal{G}$ can no longer be removed, in which case, it then returns TRUE.

Consider the example in Fig. 4(b). Suppose that we search for three-node feasible subgraphs. Since each arc has the capacity of 2, no arc will be removed due to the aforementioned rule 1. In addition, each node's in-degree is 2. Thus, nodes and arcs will be removed due to the aforementioned rule 2. In addition, both nodes v_1 and v_2 have arcs to v_0.

Procedure 4 (Searching for k-Node Feasible Subgraphs)

In the current simplified coding graph $\mathcal{G}$, pick a coding subgraph $\mathcal{G}'$ that includes node u_0 and also $k-1$ other nodes. Based on property P2, determine if there are feasible subgraphs $\mathcal{G}_f = (\mathcal{N}_f, \mathcal{A}_f)$ in this coding subgraph $\mathcal{G}'$. If so, calculate the weight $W(\mathcal{G}_f, r_n)$ of each feasible subgraph $\mathcal{G}_f$ at each rate r_n, where

$$W(\mathcal{G}_f, r_n) = \frac{s_{(u_0,u_k,P_0)} + \sum_{u_i \in \mathcal{N}_f / \{u_0\}} \left[1 - \left(1 - z^n_{(u_0)}\right)^{1/z^n_{(u_i)}}\right] s_{(u_i,u_k,P)}}{\left(\max_{(u_i,u_j,P) \in \mathcal{A}_f} s_{(u_i,u_j,P)}\right)/r_n + T_c}$$

record this feasible subgraph if its weight is currently the largest (the weight of a feasible coding subgraph is just the ETE of its corresponding feasible coding solution), and record the current transmission rate r_n. Conduct this operation for each subgraph $\mathcal{G}'$. If current coding graph $\mathcal{G}$ contains no feasible subgraph, return FALSE; otherwise, return TRUE.

Formally, the new coding algorithm is given as follows.

Joint PC-RS Algorithm
Input:
Number of available transmission rates N_r
R_i $(0 \leq i \leq M)$ and size of each packet in R_i
S_i $(0 \leq i \leq M)$
Packet delivery ratio $p_{k,i}$ $(0 \leq k \leq K, 1 \leq k \leq N_r)$
Main procedure

```
1    Construct the directed graph G by Procedure 1.
2    Execute Procedure 2 to simplify the graph G.
3    for k = 2 to min{4, node number of G} do
4        Execute Procedure 3 for G.
5        if (Procedure 3 return FALSE) then
6            go to Exit.
7        else
8            Execute Procedure 4 for G.
9        end if
10       if (Procedure 4 return FALSE) then
11           go to Exit.
12       end if
13   end for
```

Exit: Return the feasible subgraph which includes node u_0 and has the largest weight, and also its corresponding transmission rate.

It is easy to know that if there exists no i-node feasible subgraph, there definitely does not exist any $(i+1)$-node feasible subgraph. To take advantage of this property, it is in ascending order of k that we search for k-node feasible subgraphs and calculate their weights, as shown in step 3 of the proposed algorithm. Furthermore, according to property P3, the search will be conducted up to $k = 4$.

Regarding the computational complexity of the coding algorithm, we can know that Procedures 1, 2, and 3 take time $O(\max\{M^2 n, MN_r\})$, $O(M^2 n)$, and $O(M^2 n)$, respectively, where M is the number of downstream neighbors, and n is the number of flows passing through a link. Procedure 4 has the highest computational complexity $O(N_r M^3 n^3)$ when $k = 4$. Thus, the computational complexity of this algorithm is $O(N_r M^3 n^3)$. This complexity is quite low. First, N_r is clearly a small value (being 4 when using IEEE 802.11b). Second, nodes in a multihop wireless network usually have small degrees, and thus, the number of downstream neighbors M is small. Additionally, there are usually not many concurrently active flows passing a network node (i.e., n is not large), due to the very limited node transmission capacity, the load-balancing routing, etc. In addition, before starting the transmission of a data packet, a control packet (request to send) is sent to the receiver, and the receiver needs to return another control packet (clear to send). This spare time is enough to conduct the algorithm to find the coding solution and transmission rate.

In this joint coding and rate selection scheme, similar to the available COPE architecture, each network node only needs to obtain one-hop neighborhood information (link packet loss rate and the neighbors' possessed packets) from its neighbors. Thus, this scheme operates in a distributed manner using local information only. The corresponding issue we need to consider is the effective exchange of local information between neighbors. The exchange mechanism adopted in COPE is effective, but one might be able to design more effective mechanisms to exchange local information.

IV. Performance Evaluation

In this section, we investigate the network throughput improvement achieved when using the proposed rate-adaptive network-coding-based transmission scheme.

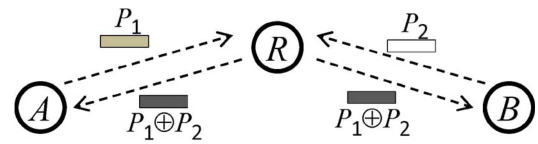
A. Simulation Setting

To evaluate the effectiveness of our proposed joint PC-RS scheme, we conducted simulations with NS-2 [18] (implementing the packet coding operation and rate-adaptive function), running 802.11b at the medium-access control (MAC) layer and ad hoc on-demand distance vector routing protocol [19] at the routing layer.

For wireless channels, we assume that the successful packet reception probability p_1 of one link (R, v_i) (with distance d) using the 1 Mb/s rate is under the lognormal shadow-fading model. Then, $p_{i,1}$ can be approximated by $p_{i,1} = 1 - 0.5(d/R)^{2\alpha}$ if $d \leq R$, $p_{i,1} = 0.5(2 - d/R)^{2\alpha}$ if $R < d \leq 2R$, and $p_{i,1} = 0$ otherwise [20], where R is the distance such that $p(d = R) = 0.5$, and α is the power attenuation factor ranging from 2 to 6. In the simulation, the path loss exponent α is set to 4, and R is set to 200. Since the packet delivery ratio usually decreases as the transmission rate increases, from $j = 2$ to 4, $p_{i,j}$ is set to $\rho \cdot p_{i,j-1}$, where ρ is a randomly generated value between 0.5 and 1.

B. Throughput Evaluation in a Three-Node Network

Both network coding and rate adaption enhance the transmission efficiency of network nodes. Thus, before investigating the network-level throughput improvement achieved by using rate-adaptive network-coding-based transmission, we first investigate the node-level throughout improvement in the well-known three-node network in Fig. 5. In this network, there exist one flow from node A to B and one flow from node B to A, with the help of intermediate relay R. The packet size is 512 B.



Packet delivery ratio

	(A, R)	**(R, B)**
1Mbps	0.9920	0.9840
2Mbps	0.7416	0.8850
5.5Mbps	0.6540	0.6810
11Mbps	0.5000	0.6240

Fig. 5. One simple three-node network and corresponding packet-delivery ratios of links at different transmission rates.

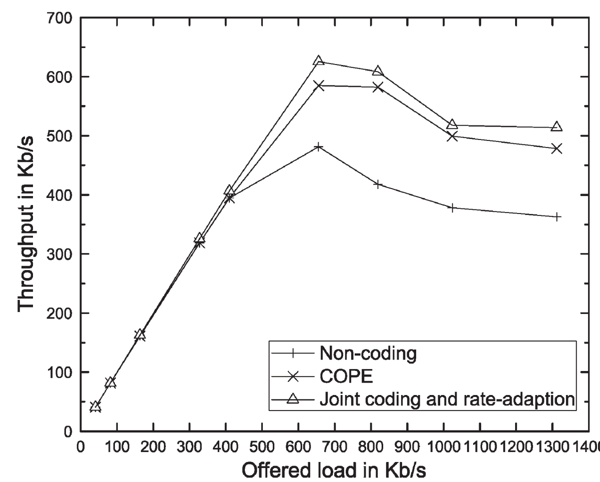


Fig. 6. Achieved throughput of different transmission schemes in the three-node network in Fig. 5.

Fig. 6 shows the throughputs achieved by the noncoding transmission scheme, COPE, and our rate-adaptive coding-based scheme under different offered loads. The x-axis in the figure represents the total rate of 30 user datagram protocol (UDP) flows in our simulation. All flows have the same rate. For the noncoding scheme and COPE with fixed transmission rate, this network achieves the maximal throughput when using the rate of 11 Mb/s. Thus, in Fig. 6, we just show these two schemes' throughputs achieved when using the rate of 11 Mb/s. In this network, R is the interaction node (i.e., bottleneck node) of two flows. It can be observed that, when the workload is small, different schemes achieve almost the same throughput. When the workload is high enough, however, these schemes demonstrate different performance. When using COPE, node R can encode two packets together to transmit (equivalently, increase its transmission rate) and thus improve the throughout as compared with the noncoding scheme. This throughput improvement is clearly reflected in Fig. 6. However, when using the rate-adaptive coding-based scheme, the throughput can further be improved, because when transmitting encoded packets from node R to A and B, the rate of 5 Mb/s can achieve higher transmission efficiency than the rate of 11 Mb/s. We will show in the next section that this improvement is significant in networks with more nodes.

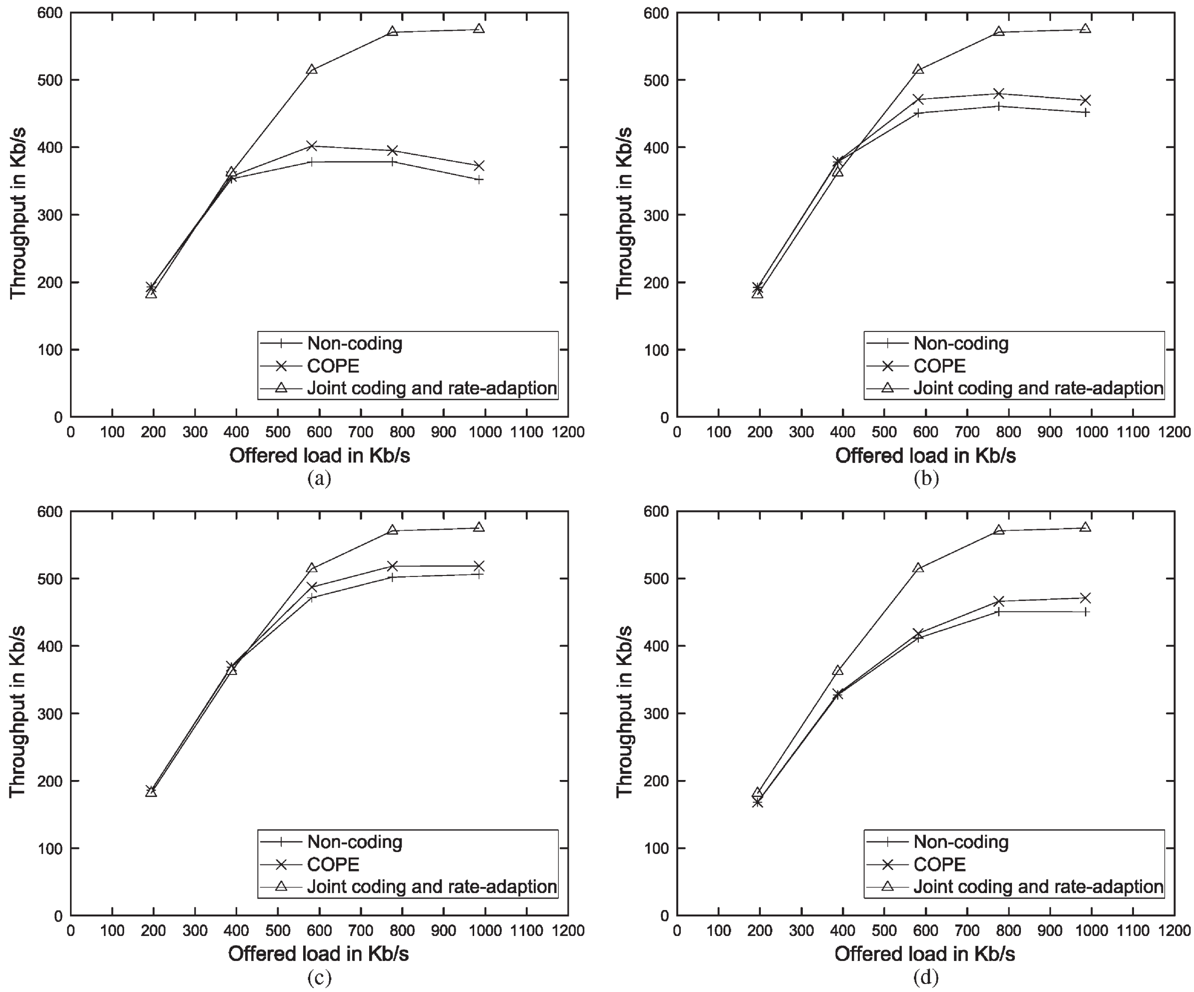


Fig. 7. Achieved throughput of different transmission schemes in the 20-node network. (a) When the noncoding transmission and COPE-based transmission use the fixed rate of 1 Mb/s. (b) When the noncoding transmission and COPE-based transmission use the fixed rate of 2 Mb/s. (c) When the noncoding transmission and COPE-based transmission use the fixed rate of 5.5 Mb/s. (d) When the noncoding transmission and COPE-based transmission use the fixed rate of 11 Mb/s.

Additionally, although network coding can essentially improve the throughput, the improvement is lower than the expectation. When using COPE, ideally, node R can always transmit the encoded packet and double the throughput. We can see, however, that the throughput is improved by about 30%. This is because two important factors prohibit the achievement of this ideal throughput enhancement. The first one lies in the medium-access mechanism. If the medium-access opportunities are always allocated in the following order: nodes A, B, and R, then R always has packets of two flows in its buffer and, thus, can always encode two packets destined to different receivers. Due to the random access mechanism of 802.11 MAC, however, from time to time, only one flow's packets exist in node R's buffer, which is clearly reflected in the simulation process. Thus, node R will frequently transmit native packets rather than the encoded packet. The second factor is the packet loss. It frequently happens that when $P_1 \oplus P_2$ is successfully received by its designated receiver (e.g., A) after one or multiple transmissions at R, node B does not yet receive this encoded packet. In this case, it is equivalent to the case that node R just transmits native packet P_2 to A. One can easily know that this case will even become worse when the size of P_1 is larger than P_2.

C. Throughput Evaluation in a 20-Node Network

Now, we investigate the throughput achieved by different transmission schemes in a more general network with 20 nodes, whose positions are randomly generated inside the region $400 \times 400\ \mathrm{m}^2$.

For each transmission scheme, we evaluate its corresponding throughputs in ten different cases. In each case, there are 30 UDP flows whose source and destination are randomly selected. The data packet size of each flow remains unchanged, and packet sizes of different flows follow the packet-size distribution presented in [17]. We simulate the achieved throughput in each case and average the achieved throughputs over these ten cases.

Fig. 7 shows the throughputs obtained by the noncoding scheme, COPE, and the network-coding-based rate-adaptive scheme. From its four subfigures, first, we can observe that COPE only slightly improves the throughput, as compared with the traditional noncoding transmission. There are several reasons for this. First, the number of nodes with coding opportunities (i.e., the interaction nodes of multiple flows) is limited. They are mainly located at the center area of the network region. Second, it is a rare case that three or more packets are encoded

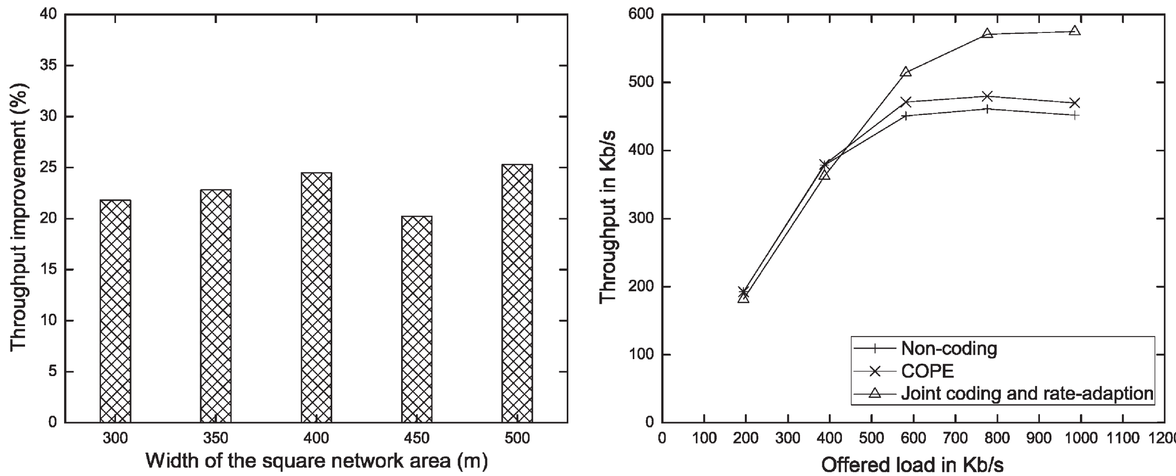


Fig. 8. Throughput improvement achieved by our proposed scheme as compared with COPE under different densities and different numbers of flows. (a) Average throughput improvement under different densities. (b) Average throughput improvement under different numbers of flows.

together. Additionally, most flows need to traverse multiple hops before reaching the destination node. For an arbitrary flow, although the transmission efficiencies of some nodes on its route may be improved, the throughput is likely to slightly improve because the bottleneck node of this flow has no coding opportunities.

Second, we can clearly see that, compared to COPE, our proposed scheme can greatly improve the network throughput. This improvement comes from the joint adoption of better coding operations and the rate-adaptive transmission. On one hand, from the coding perspective, the proposed scheme adopts an efficiency-oriented metric to select the coding solutions with high transmission efficiency. On the other hand, from the adaptive-rate point of view, our rate-adaptive transmission effectively takes not only the packet loss probability but the transmission time into account as well to select an appropriate transmission rate. The rate-adaptive transmission scheme effectively enhances all network nodes' transmission efficiency, rather than limited nodes' transmission efficiency, as in COPE.

We now investigate the performance of our proposed scheme under different densities and different numbers of flows. Fig. 8 shows the throughput improvement achieved by our scheme as compared with COPE. The results in Fig. 8(a) are obtained under different network area sizes. The number of nodes (20 nodes), the number of flows (30 flows), and the nodes' relative positions are the same as those introduced at the beginning of this section. From this subfigure, we can see that there is no clear increasing/decreasing trend as the node density decreases, and the throughput improvement is always significant. This can be explained as follows. Compared to COPE, the throughput improvement achieved by the proposed scheme comes from both the rate adaption and better utilization of coding opportunities. As the node density decreases, the packet-coding opportunities decrease (due to less successful overhearing), and thus, the gain from better utilization of coding opportunities decreases, but the difference in link quality becomes significant, and the gain from transmission rate selection increases. The results in Fig. 8(b) are obtained under different numbers of flows in the same network introduced at the beginning of this section. From this subfigure, it can be observed that, as the number of flows increases, the throughput improvement also increases. It is notable that the coding opportunities come from the intersection of different flows. As the number of flows increases, the coding opportunities increase, and thus, the throughput improvement from the better utilization of coding opportunities also increases.

In the aforementioned performance evaluation, we only investigated the throughput improvement in the case of using IEEE 802.11b, which supports four different transmission rates. When the optional rates are numerous, it can be expected that the performance improvement will become more significant due to the significant quality difference of network links. This remains as future work.

V. Conclusion

In this paper, we have applied the rate-adaptive transmission mechanism in network-coding-based multihop wireless networks. We have first shown that there exists an interaction between coding operation and rate selection and then proposed an efficient algorithm for jointly selecting good coding solutions and their corresponding transmission rates. Simulation results have demonstrated that, compared with the current coding-based transmission scheme with fixed transmission rate, our rate-adaptive transmission scheme can significantly improve the network throughput. This improvement can be as large as 50%.

Notice that the proposed metric of ETE only considers node transmission efficiency and does not directly reflect the network performance. The determination of coding operation and rate selection based on more information like some global information might more effectively improve the network performance. Therefore, one future work is to design a more intelligent joint coding operation and rate selection scheme with the utilization of more network information. Additionally, in wireless ad hoc networks with high mobility, the link quality rapidly changes due to node movement. In such networks, the joint packet operation and rate selection is much more complicated and is worth our further efforts. There are two ways to go about doing joint coding operation and rate selection. The first way is still using the ETE metric but needs to timely and accurately predict the packet-loss rates at neighbors based on some short-term information. For example, we predict the SNR of one link based on the relative movement speed and then build the relationship between the SNR and the packet-loss rate based on the characteristics of a specific network. The second way is to utilize a long-term performance measure that depends on the channel statistics.

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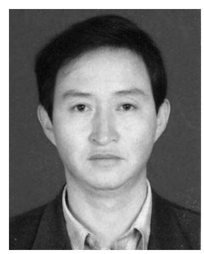

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