Effects of circular motion on judgment of rotation direction and depth order in visual motion

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Abstract

The rotation direction and depth order of a rotating sphere consisting of random dots often reverses while it is viewed under orthographic projection. However, if a short viewing distance is simulated under perspective projection, the correct rotation direction can be perceived. There are two motion cues for the rotation direction and depth order. One is the speed cue; points with higher velocities are closer to the observer. The other is the vertical motion cue; vertical motion is induced when the dots recede from or approach the observer. It was examined whether circular motion, which does not have any depth information but induces vertical velocities, masks the vertical motion cue. In Experiment 1, the effects of circular motion on the judgment of the rotation direction of a rotating sphere were examined. The magnitude of the two cues (the speed cue and the vertical velocity cue) as well as the angular speed of circular motion was varied. It was found that the performance improved as the vertical velocity increased and that the speed cue had slight effects on the judgment of the rotation direction. It was also found that the performance worsened as the angular speed of the circular motion was increased. In Experiment 2, the effects of circular motion on depth judgment of a rotating half sphere were investigated. The performance worsened as the angular speed of the circular motion increased, as in Experiment 1. These results suggest that the visual system cannot compensate for circular motion perfectly for the judgment of the rotation direction and depth order.

1. Introduction

In studies on perception of structure from motion (SFM), moving images were often generated by orthographic projection. Computationally, the rotation direction and depth order cannot be uniquely determined based on image motion alone. This is because the image motion of an object generated by orthographic projection is the same as that of an object reversely rotating in mirror-reflected depth. There are two solutions for moving images viewed under orthographic projection. When observing the orthographic projection of a random-dot object rotating around an axis parallel to the image plane, the rotation direction and depth order of the object appear to be reversed.

On the other hand, theoretically, the rotation direction and depth order can be recovered from the motion viewed under perspective projection. It was reported that human observers perceive the correct depth order (i.e., convex or concave) or rotation direction when they viewed perspectively projected motion images at short viewing distances (Braunstein, 1977; Dijkstra, Cornilleau-Peres & Droulez, 1995; Hershberger, Stewart & Laughlin, 1976; Rogers & Rogers, 1992). This implies that the human visual system is able to disambiguate depth order and rotation direction in some ways.

Several cues for depth order and rotation direction have been suggested (Braunstein, 1977; Hershberger, Carpenter, Starzec & Laughlin, 1974; Hershberger & Urban, 1970; Rogers & Rogers, 1992). Braunstein (1977) reported that one of the most efficient cues is vertical perspective. When an object is rotating around a vertical axis passing through it, the perspectively projected vertical length of a part of the object reduces when that part is receding, and increases on approach; this induces vertical velocities. The vertical velocities facilitate the disambiguation of the rotation direction and depth order.

When an object rotates around an axis slanted away from or toward the observer, divergent motion is added to the flow filed. Further, when an observer rotates around her/his line of sight (for example, inclines his/her head), circular motion is added to the flow field. As a consequence, different vertical velocity components are included in the flow, and the vertical velocities do not directly indicate the rotation direction. Some researchers have suggested that the circular velocity components of a flow filed are compensated for in the visual system (Todd & Bressan, 1990; Lind, 1996). There exists some experimental evidence that the human visual system compensates for circular motion for rigidity judgment in SFM (Todd & Bressan, 1990). However, whether the visual system can compensate for circular velocity components for the judgment of rotation direction and depth order has not been examined. In this study, we investigated the effects of circular motion on the judgments of rotation direction and depth order in Experiments 1 and 2, respectively.

Some studies have suggested that human observers disambiguate the depth order in SFM using speed information (Braunstein, 1977; Hershberger & Urban, 1970); objects moving at higher velocities are perceived to be closer to the observer. Under perspective projection, the retinal speed of a point rotating around an axis is higher when it is closer to the rotation axis, and closer objects move faster in the case of horizontal translation of the objects or the observer. It has been reported that the speed cue is actually used for the disambiguation of the rotation direction or depth order (Braunstein, Liter & Tittle, 1993; Braunstein & Tittle, 1988; Hershberger & Urban, 1970). The effectiveness of the cue, however, has not yet been clarified; the effectiveness values somewhat varied in different studies. In this study, we have also investigated how effectively human observers use this cue.

In order to examine the effectiveness of the speed cue, we used two flow types: one included the speed cue (flow type I) and the other did not (flow type II). Flow type I was the flow generated by perspective projection. In flow type II, the horizontal velocity was generated by orthographic projection. (Since the degree of perspective for the vertical velocity was varied, the vertical velocity was not necessarily consistent with the perspective projection.) Comparing the performance for the two flow types, we measured the effectiveness of the speed cue for the judgment of the rotation direction and depth order.

Computationally, the depth order can be recovered when the rotation direction is known and vice versa. However, the perception of the depth order might not be directly linked with the perception of the rotation direction. We also compared the performance of the depth judgment and that of the rotation direction judgment, and examined whether the judgment of the depth order was consistent with that of the rotation direction.

2. Experiment 1: Judgment of the rotation direction

Although rotation around the line of sight, termed roll, is usually small in daily life, roll velocity components are included when an observer's head is inclined. (Since the retinal flow depends solely on the relative movement between an observer and an object, the same flow arises when the object rotates around an axis slanted away from or toward the vertical axis.) Although circular velocities due to roll hold no depth information, vertical velocities are caused by roll. If fast circular motion was included in the flow, the vertical perspective cue becomes unreliable. Hence, the performance of the rotation direction judgment should worsen with circular motion. However, the human visual system may be able to deal with circular motion by removing the circular velocities before employing the vertical motion cue. We examined the effects of the roll velocity components on the perception of the rotation direction of a rotating random-dot sphere. Further, we examined the effectiveness of the speed cue for the judgment of the rotation direction.

1. Methods

Apparatus. The stimuli were generated by an AT compatible computer and displayed on a CRT display. The observers viewed the stimuli monocularly with their head supported on a chin rest. The viewing distance was 35 cm. The refresh rate of the display was 75 Hz, and the display size was 1024 pixels × 768 pixels, subtending 46 deg × 36 deg.

Observers. Seven observers participated in this experiment. One was the author. The others were unaware of the purpose of the experiment. All the observers had normal or corrected-normal acuity.

Stimuli. The stimuli were images of 400 moving white dots configured on the surface of a sphere. We moved the dots by simulating situations where the sphere was rotating around an axis. (It should be noted that as long as the movement of an object relative to that of an observer is the same, the same flow will arise. This implies that the same flow will arise when an observer moves around the sphere. When the stimulus is viewed, the observers do not feel that they are moving, but perceive the sphere to be rotating. This may be due to the moderate size of the stimulus area and the lack of other information on self-motion such as vestibular information.) Both the front and back surfaces were presented; the back surface was visible through the front random-dot surface, which was transparent. The size of the dots was 3 pixels × 3 pixels. We simulated a sphere with a radius of 8 cm whose center was located at a distance of 35 cm from the observers. The dots were uniformly distributed in an aperture with a radius of 13.2 deg. In order to focus on the motion cue and avoid the use of path cues, the lifetime of the dots was limited to 16 frames (213 ms). At the beginning of the stimulus presentation, the lifetime of the dots was determined at random and ranged from 1 to 16. The dots were displayed for the duration of their lifetime and disappeared after their lifetime elapsed; new dots were then presented at random positions within the aperture. Whether the dots were displayed on the front or the back surface was determined at random. The presentation duration was 1.0 s. If a longer presentation duration had been employed, depth reversal would have occurred and the judgment would have been difficult. Hence, we chose a rather short stimulus duration. It has been shown that a presentation duration of 1.0 s is sufficient to distinctly perceive structure from motion (Hildreth, Grzywacz, Adelson & Inada, 1990; Eby, 1992). In addition, we obtained essentially the same results when a presentation duration of 2.0 s was employed for an observer. (However, these results are not provided in this paper.)

Two motion flow types (flow type I and flow type II) were used. (See Appendix for details on the motion patterns of the dots.) The motion for flow type I was almost equal to the perspectively projected flow of the random-dot sphere rotating around an axis passing through its center. For flow type II, the orthogonal projection was simulated for the horizontal velocities. For flow type I, the speed cue was included; the dot with the maximum speed was displayed on the front surface, and the rotation direction could be obtained from the motion direction of the dot. On the other hand, the stimuli did not include the speed cue for flow type II.

Figure 1. Insert the figure about here.

The simulated rotation rate around the vertical axis was 0.57 rad/s (32.7 deg/s). (The sign of the rotation rate indicates direction.) We added circular motion (roll velocity components) to the flow. We used five roll rates: -30, -15, 0, 15, and 30 deg/sec.

The vertical velocities of the stimuli were multiplied by a parameter, which controlled the magnitude of the vertical motion cue (See Appendix for details). We used four values of g_p: 0, 0.5, 1.0, and 2.0. The case $g_p = 0$ indicates no vertical velocity; $g_p = 1.0$ indicates the original vertical velocity; and $g_p = 0.5$ and 2.0 indicates half and twice the vertical velocity, respectively. Examples of the dot paths of the stimuli are shown in Fig. 1.

Procedure. The observers' task was to judge the rotation direction of the rotating sphere. They were asked to indicate the motion direction of the front surface by pressing either the right or the left button on a gamepad. When they did not perceive rotation (for example, they perceived two slipping surfaces), they were instructed to indicate the rotation direction that they perceived more strongly.

All the variables were varied across sessions. There were 80 stimulus conditions: two flow types × four values of g_p × five roll rates × two rotation directions. Five trials were run for each stimulus condition in a single session. The observers participated in four sessions. Each session had a duration of about 20 min. The observers underwent one short practice session before the experimental sessions. No feedback was given in both the practice and the experimental sessions.

2. Results and discussion

The stimuli used in this experiment do not occur in any actual situation. Hence, it was not possible to determine the correct responses, especially when $g_p = 0$ and flow type II was used (i.e., when orthographic projection was used). We defined the correct response as the response consistent with the sign of the simulated rotation rate around the vertical axis, which was used for the stimulus generation. For the calculation of the percentage of correct responses, the data were collapsed across the rotation directions.

The average percentages of correct responses for all the observers are shown in Fig. 2. It was found that as g_p increased, the percentage of correct responses increased for both the types of flow. The results clearly indicate that the observers used the vertical motion cue for the judgment. For g_p =2.0 and when no roll was simulated, the percentage of correct responses exceeded 80%. As the simulated roll increased, the performance worsened. The decline in the performance was clear for g_p =1.0 and 2.0. The performance for flow type I did not greatly differ from that for flow type II, which suggests that the effect of the speed cue was negligible.

A repeated-measure three-way (two flow types × four values of g_p × five roll rates) analysis of variance was conducted. The main effect of the roll rate ($F(4, 24)=11.8$, $p<.01$) as well as that of g_p (the vertical motion cue) ($F(3, 18)=31.6$, $p<.01$) were significant, while the effect of the flow type ($F(1, 6)=5.7$, $p=.054$) was marginally significant. Further, the interaction between the roll rate and the vertical motion cue ($F(12, 72)=4.4$, $p<.01$) was significant, but the other interactions were not significant. Further, the simple main effects of the roll rate were significant for g_p =2.0 ($F(4, 96)=18.9$, $p<.01$) and g_p =1.0 ($F(4, 96)=8.4$, $p<.01$). Ryan's post-hoc tests showed that, for g_p =2.0, all the differences between the four roll rates except those between ± 30 deg/s, ± 15 deg/s, and 0 and 15 deg/s were significant; for g_p =1.0, the differences between 0 and 30 deg/s, 0 and -30 deg/s, -15 and 30 deg/s, and 15 and 30 deg/s were significant. The simple main effects of the roll rate for g_p =0.5 and for g_p =0 were not significant.

Figure 2. Insert the figure about here.

These results suggest that the observers applied the vertical motion cue to the rotation axis for the judgment of the rotation direction. The effects of the speed cue were unreliable, and the cue was not as effective as the vertical motion cue. The decline in the performance with the increase in the roll rate indicates that the visual system cannot perfectly compensate for the roll velocity components for the judgment of the rotation direction when it uses the vertical motion cue.

3. Experiment 2: Convex/Concave Judgment

In Experiment 1, we examined the perception of the rotation direction of a random-dot sphere. Computationally, the depth order can be recovered when the rotation direction is known and vice versa. However, the perception of depth order may not be directly linked with the perception of rotation direction. In this experiment, we examine the effects of the roll velocity components on the concave/convex judgment.

1. Methods

The simulated object was half of the sphere used in Experiment 1. The stimulus was generated in the same way as in Experiment 1, except that the dots were displayed on either the front or the back surface. When the dots on the front surface were displayed, the convex surface was presented, while when the farther dots were displayed, the concave surface was presented. The number of dots was statistically half of that used in Experiment 1 (approximately 200 dots). It should be noted that the speed cue was ineffective for this stimulus; the fastest dot was not necessarily closer because either the back or the front surface of the sphere was displayed. The observers were asked to indicate whether the presented surface was convex or concave. The simulated speed of the rotation (i.e., B) was jittered around .57 rad/s (32.7 deg/s) by 20% to vary the average and maximum speeds of the dots between trials. Without jittering, the average and maximum speeds would be lower for flow type I of a concave surface and higher for flow type I of a convex surface, as compared to the other conditions.

Except for the above, the procedure and stimulus conditions were the same as those of Experiment 1. There were 160 stimulus conditions: two flow types × four values of g_p × five roll rates × two rotation directions × two depths (convex/concave). Two trials were run for each stimulus condition in a single session. Thus, there were 320 trials in a session. Five sessions were repeated for each observer.

2. Results and discussion

We calculated the percentage of correct responses. The data were collapsed across the depths and the rotation directions. Although we could not determine which response was correct, we defined the correct response as the response consistent with the simulated depths. The results are shown in Fig. 3. The performance level was similar to that of Experiment 1. The performance of the convex/concave judgment worsened as the roll rate increased.

A repeated-measure three-way (two flow types × four values of g_p × five roll rates) analysis of variance was conducted. The main effect of the roll rate ($F(4, 24)=22.6$, $p<.01$) as well as that of g_p (the vertical motion cue) ($F(3, 18)=111$, $p<.01$) were significant. The effect of the flow type ($F(1, 6)=.13$) was not significant. The interaction between the roll rate and the vertical motion cue ($F(12, 72)=6.9$, $p<.01$) was significant; however, all the remaining interactions were not significant. Further, the simple main effects of the roll rate were significant for g_p =2.0 ($F(4, 96)=23.1$, $p<.01$), g_p =1.0 ($F(4, 96)=24.6$, $p<.01$), and g_p =0.5 ($F(4, 96)=12.6$, $p<.01$). Ryan's post-hoc tests showed that, for g_p =2.0 and 1.0, all the differences between the four roll rates except those between ± 30 deg/s and between ± 15 deg/s were significant; for g_p =0.5, the differences between 0 and 30 deg/s, 0 and -30 deg/s, -15 and 30 deg/s, 0 and 15 deg/s, and 0 and 15 deg/s were significant. The simple main effect of the roll rate for g_p =0 ($F(4, 96)<1$) was not significant.

These results suggest that the observers used the vertical motion cue for the judgment of the rotation direction. The flow type had a slight effect on the performance. This result is reasonable because the speed cue is ineffective for a half sphere. The decline in performance with the increase in roll rate indicates that the visual system cannot perfectly compensate for the roll velocity components for the judgment of the depth order when it uses the vertical motion cue.

The results in Experiment 2 show a trend similar to that of Experiment 1, which suggests that the perceived depth was consistent with the perceived rotation direction.

Figure 3. Insert the figure about here.

4. General discussion

The findings of this study are (1) the roll velocity components of the retinal velocity field degrade the judgment of the rotation direction and depth order, (2) the speed cue had marginal effects on the judgment of the rotation direction, and (3) the performance of the depth judgment is very similar to that of the rotation direction judgment. Given below are the independent discussions on these findings.

1. Compensation for roll

Todd and Bressan (1990) and Lind (1996) proposed simple methods for estimating roll for orthographically or perspectively projected motion, and suggested compensation mechanisms for roll components of the motion field in the visual system. However, the results of Experiments 1 and 2 imply that the human visual system cannot compensate for the roll velocity components perfectly for the judgment of the rotation direction and depth order.

The performance for the roll rates of ±30 deg/sec and = 1.0 or 2.0 were above chance level. This may suggest that the roll velocity components are compensated for to some extent. However, it may be unnecessary to assume a compensation mechanism for the roll velocity components in order to explain the results in Experiments 1 and 2. The average of the velocities due to roll across all the images is zero. If all the vertical velocities are pooled (or summed up), we can obtain the correct rotation direction and depth order. The performance above chance level for roll rates of ±30 deg/sec and = 1.0 or 2.0 can be explained by the pooling mechanism.

Although the decline in performance with the increase in roll rate is explained with the absence of the compensation mechanism, the former phenomenon has an alternative explanation - noise is enhanced by roll velocities. The average speed is greater for ± 30 deg rolls than for no roll. An increase in speed might amplify noise due to multiplicative noise. However, noise amplification by circular motion is rather implausible. The increase in the average speed from no roll to a roll rate of 30 deg/s was approximately 35% for the stimulus in Experiment 1. Considering that the visual system can detect motion for a range of at least 1 arcmin/s to 100 deg/s (De Bruyn & Orban, 1988; Shioiri, Ito, Sakurai & Yaguchi, 2002), the change in speed is quite small, and the human performance for motion perception should be invariable with this small change in speed. Moreover, the speed of a number of dots was decreased by roll; however, this number was smaller than the number of dots whose speed was increased by roll. The jittering of the rate of rotation around the vertical axis in Experiment 2 causes a slight difference in performance between Experiments 1 and 2.

Todd and Bressan (1990) presented experimental evidence that the human visual system compensates for circular motion for rigidity judgment in SFM. Further, Perotti, Todd, Lappin, and Phillips (1998) examined human perception of curvature judgment from motion and the masking effects of a global motion pattern of curl (i.e., circular motion). The task of the observers in their study was to adjust the two principal curvatures of the stimuli. It was found that the addition of circular motion does not affect the judgment of the shape characteristic (ratio of the two principal curvatures) although it lowered the magnitude of the perceived curvedness (square root of the mean squares of the two principal curvatures) very slightly, i.e., the perceived surface became slightly flatter.

On the other hand, the results in Experiments 1 and 2 show that the human visual system cannot compensate for the roll velocity components perfectly for the judgment of the rotation direction and depth order. The difference between our results and those of Todd and Bressan (1990) and Perotti et al. (1998) can be attributed to the difference in the task; for different tasks, different cues and strategies are employed.

The vertical velocity is a significant cue for the judgment of the rotation direction and depth order. The circular motion induces an additional vertical velocity. The decline in performance due to circular motion indicates that the visual system uses the vertical velocities including the roll velocity components. On the other hand, it is suggested that for curvature detection from visual motion, the visual system uses spin variation (Droulez & Cornilleau-Peres, 1990), which is invariable irrespective of roll rates. Hence, circular motion should not affect curvature perception, as shown by Perotti et al. (1998). In addition, the visual system may perform rigidity judgment using some cues such as def (Domini, Caudek & Proffitt, 1997), which is a first-order component of the optic flow and is independent of circular motion (Koenderink & van Doorn, 1975). The difference in the effects of circular motion on the performance between the tasks suggests that these findings should not be generalized for other tasks.

2. Speed cue

Braunstein (1977) showed that, for a rotating random-dot sphere, the vertical perspective cue is used, while the speed cue is not used significantly. In another study, however, he and his coworkers showed that human observers used the sign of the velocity gradient for the depth order judgment of a hinge-shaped random-dot object (Braunstein et al., 1993; Braunstein & Tittle, 1988). (The sign of the velocity gradient can be categorized as a speed cue.) We have shown that the vertical motion cue is much more efficient than the speed cue, which is consistent with results of a previous report. The speed cue appears to be efficient under limited conditions; for example, it is efficient for a hinge-shaped random-dot object and not for a rotating random-dot sphere. Further studies may be required to determine the conditions under which the speed cue is used in the visual system.

3. Coupling of perception of the depth order and rotation direction

For a rotating sphere composed of random dots, the perceived rotation direction often reverses when viewing the stimulus, and simultaneously, the depth order also reverses. This may demonstrate the coupling of the perception of rotation direction and depth order. However, systematic examinations on whether they are actually coupled have not yet been performed. The results of Experiment 1 (shown in Fig. 2) were very similar to those of Experiment 2 (shown in Fig. 2). The similarity implies that the perceived depth is consistent with the perceived rotation direction. To clarify this similarity, in Fig. 4, the average percentages of the correct responses for the rotation direction judgment of Experiment 1 are plotted against those of the correct responses for the depth judgment of Experiment 2. The data points are scattered around a line with a slope of 1, and the performance of the judgment of the depth is highly correlated with that for the rotation direction. The correlation coefficient is 0.84, which is very high considering the differences in the stimulus conditions (the number of dots, the simulated shape, and the jittering of the simulated rotation speed) of the experiments. This suggests that the perception of depth order is coupled with the perception of rotation direction.

Figure 4. Insert the figure about here.

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5. Appendix

We use a coordinate system that is fixed with respect to the observer, with the axis directed along the optical axis. The [EQUATION] and [EQUATION] axes are horizontal and vertical, respectively. The observer is located at the origin. If point [EQUATION] on a sphere whose center lies on the [EQUATION] axis is rotating around the [EQUATION] axis, its movement can be expressed by the following differential equations.

[EQUATION] (1)
[EQUATION] (2)
[EQUATION] (3)

Here, [EQUATION] is the distance of the sphere center from the observer and [EQUATION] is the angular velocity of the rotation of the sphere; the sign of [EQUATION] indicates the rotation direction. If we consider the perspective projection of the velocity on an image plane with a focal length of 1 for the projection, the projected point [EQUATION] for point is given by the following equations.

[EQUATION] (4)
[EQUATION] (5)

The velocity projected [EQUATION] on the image plane is given as follows.

[EQUATION] (6)
[EQUATION] (7)

The above equations are basic flow equations for a sphere rotating around a vertical axis. We generated our stimuli as follows.
The term [EQUATION] is unique to perspective projection and does not appear for orthogonal projection. Since it is negligible for small [EQUATION], as in our experiments, we have omitted it.

[EQUATION] (8)
[EQUATION] (9)

Vertical velocity [EQUATION] is dependent on [EQUATION]; thus, we can recover [EQUATION] from it. We controlled the magnitude of the velocity component using [EQUATION].

[EQUATION] (10)

We then added the circular velocity [EQUATION] to [EQUATION].

[EQUATION] (11)
[EQUATION] (12)

Here, [EQUATION] is the angular velocity of the circular motion. It should be noted that to obtain [EQUATION], [EQUATION] was subtracted from [EQUATION]. Replacing [EQUATION] in Eq. (3) with [EQUATION] using Eq. (4), we obtain

[EQUATION] (13)

We moved the dots on the sphere according to the differential equations (11), (12) and (13). We term this motion as flow type I.

Let [EQUATION] be [EQUATION]. The inverse of depth [EQUATION] was linearly approximated around [EQUATION] .

[EQUATION] (14)

We substituted [EQUATION] and [EQUATION] in Eqs. (11), (12) and (13) with ([EQUATION] and [EQUATION], respectively

[EQUATION] (15)

[EQUATION] (16)

[EQUATION] (17)

For flow type II, we moved the dots according to these equations. The horizontal velocity [EQUATION] is equal to that generated by orthographic projection. When [EQUATION], this flow is equivalent to the flow generated by orthographic projection, and the dot moves along a horizontal line. When [EQUATION] is non-zero, the path of the dots is elliptic.

Figure captions

- Figure 1. Paths of stimuli under four conditions in Experiment 1. (a) Speed difference cue, velocity component parallel to the rotation axis (b) No speed difference cue, velocity component parallel to the rotation axis (c) Speed difference cue, no velocity component parallel to the rotation axis (d) No speed difference cue, no velocity component parallel to the rotation axis (e) An example of stimuli with roll velocity components
- Figure 2. Results of Experiment 1. The horizontal axis shows the roll rate and the vertical axis indicates the percentage of correct responses for the rotation direction. Data for four magnitudes of the vertical motion cue (g_p=0, 0.5, 1.0 and 2.0) were shown. (a) Results for flow type I. (b) Results for flow type II.
- Figure 3. Results of Experiment 2. The horizontal axis shows the roll rate and the vertical axis indicates the percentage of correct responses for the depth. Data for four magnitudes of the vertical motion cue (g_p =0, 0.5, 1.0 and 2.0) were shown. (a) Results for flow type I. (b) Results for flow type II.
- Figure 4. The average percentages of the correct responses for the rotation direction judgment of Experiment 1 for each condition are plotted against those of the depth judgment of Experiment 2.

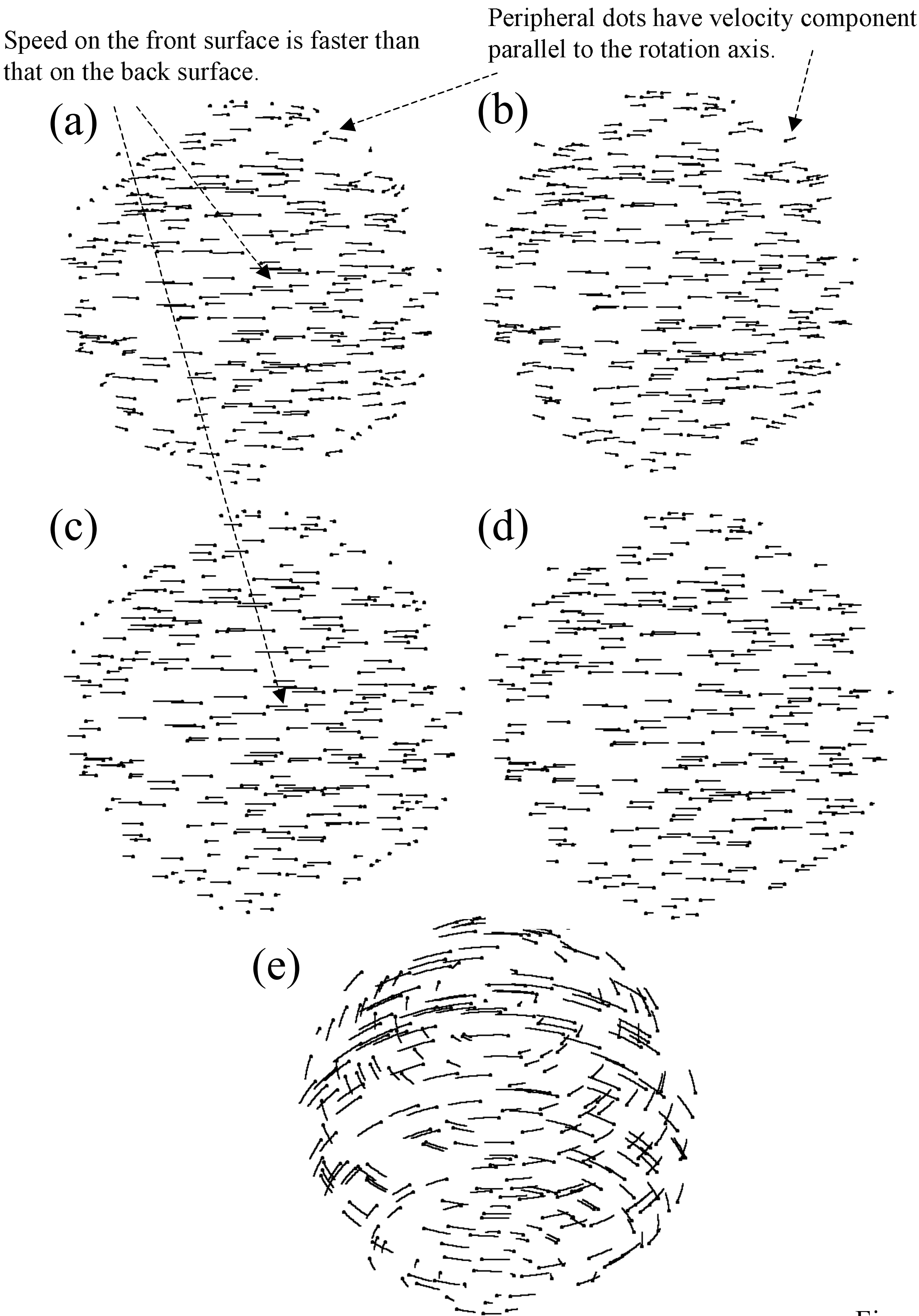
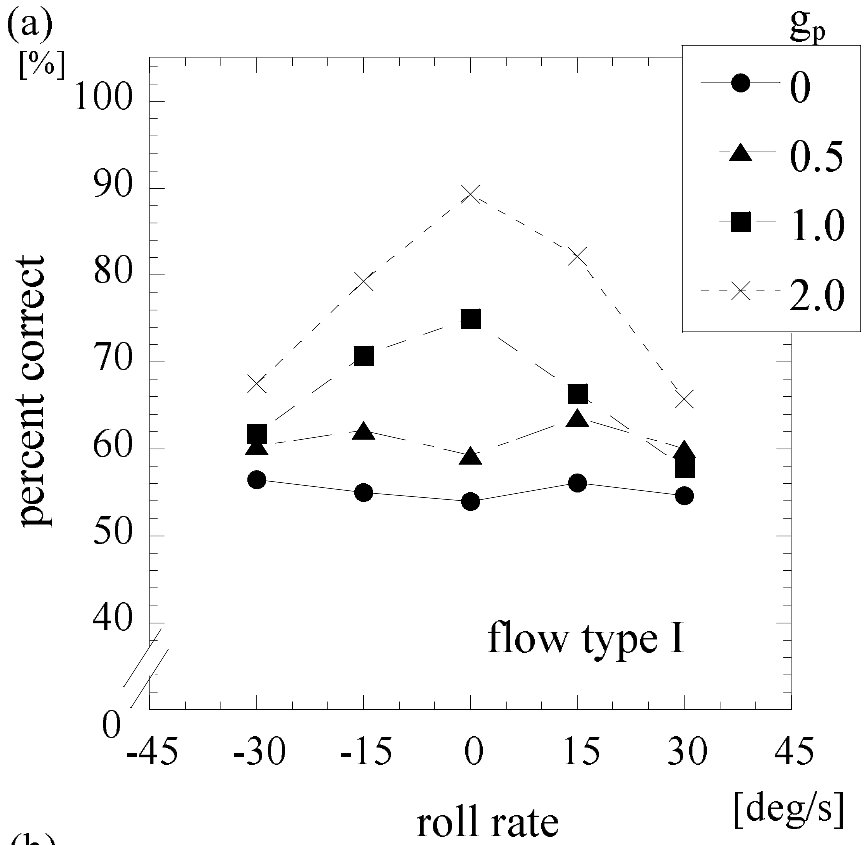
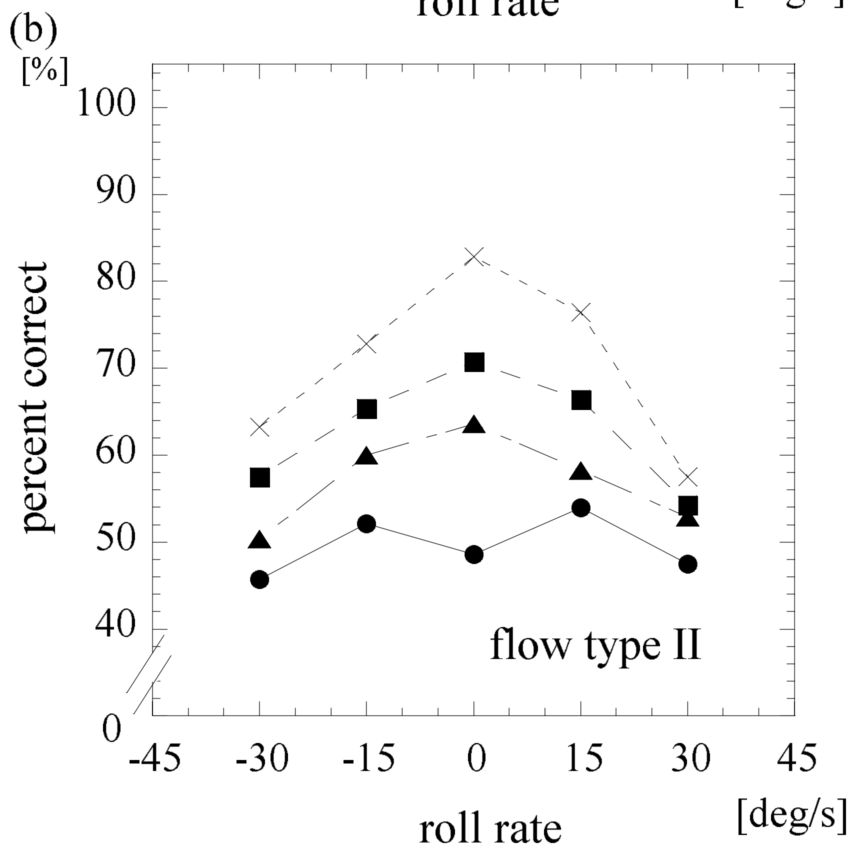

Speed on the front surface is faster than that on the back surface.
Peripheral dots have velocity component parallel to the rotation axis.
(a)
(b)
(c)
(d)
(e)

Figure 1



(a)
[%]
gp
0
0.5
1.0
2.0
100
90
80
70
60
50
40
0
percent correct
flow type I
-45
-30
-15
0
15
30
45
roll rate
[deg/s]



(b)
[%]
100
90
80
70
60
50
40
0
percent correct
flow type II
-45
-30
-15
0
15
30
45
roll rate
[deg/s]

Figure 2

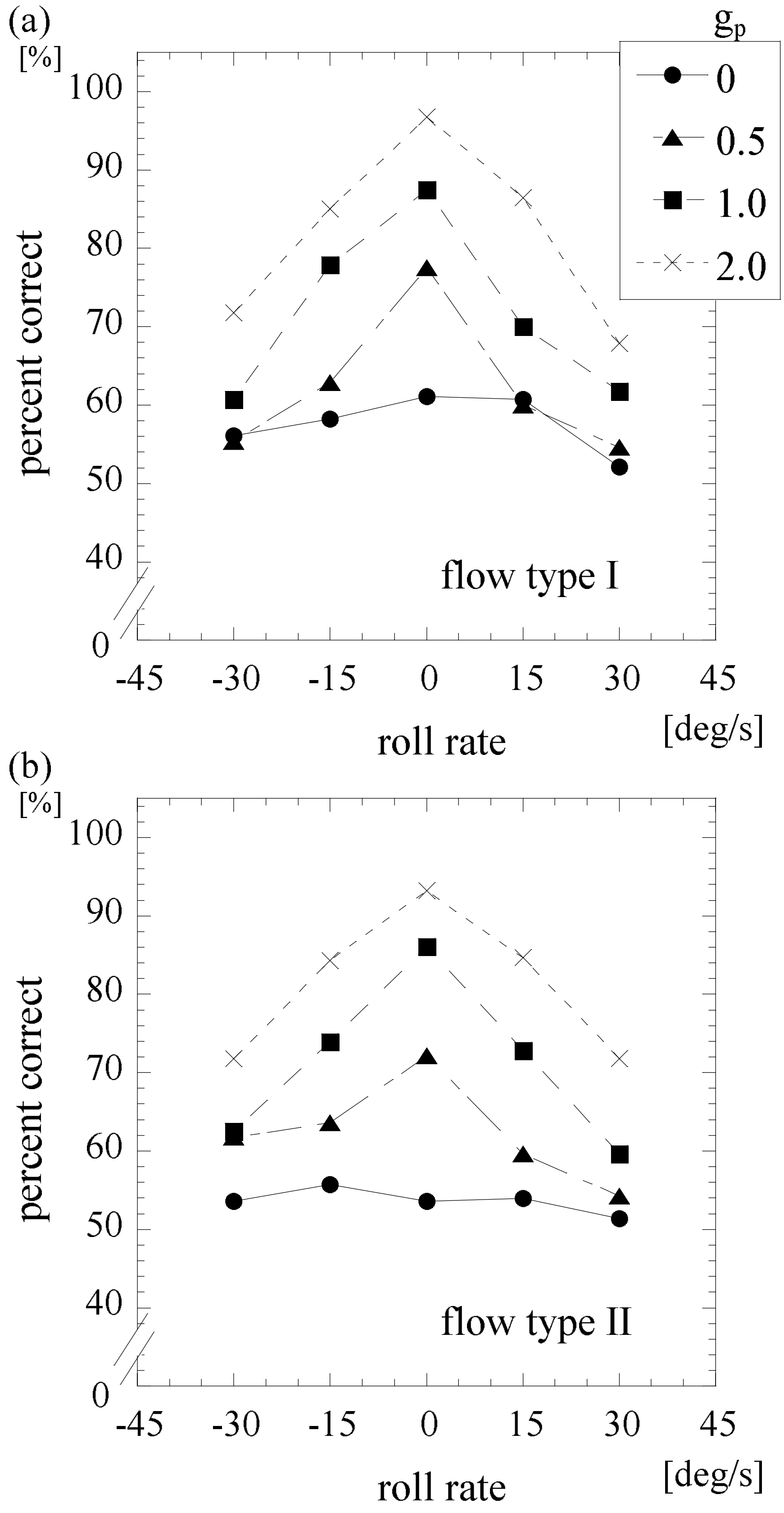
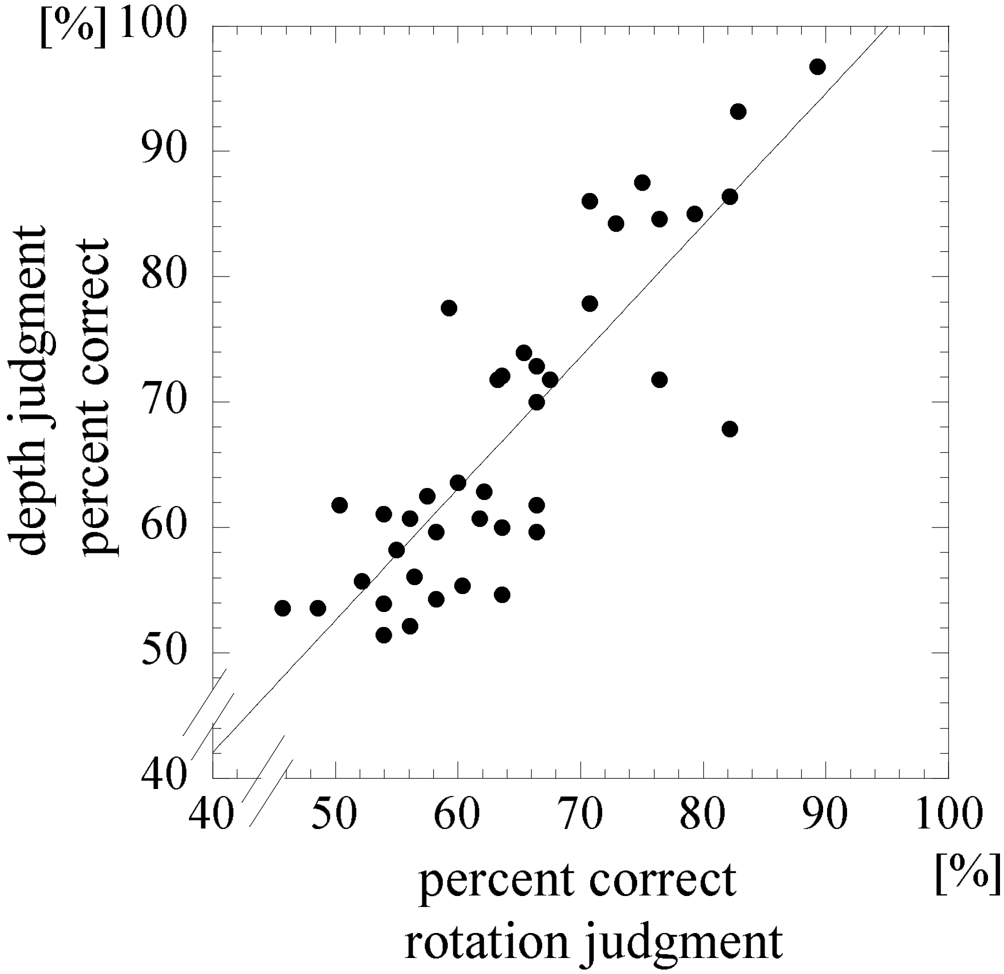

(a)
[%]
percent correct
100
90
80
70
60
50
40
0
-45
-30
-15
0
15
30
45
roll rate
[deg/s]
flow type I
g_p
0
0.5
1.0
2.0
(b)
[%]
percent correct
100
90
80
70
60
50
40
0
-45
-30
-15
0
15
30
45
roll rate
[deg/s]
flow type II

Figure 3



[%] 100
90
80
70
60
50
40
depth judgment
percent correct
40
50
60
70
80
90
100
[%]
percent correct
rotation judgment

Figure 4