Overall Blocking Behavior Analysis of General Banyan-Based Optical Switching Networks

Chen Yu, Xiaohong Jiang, Member, IEEE, Susumu Horiguchi, Senior Member, IEEE, and Minyi Guo, Member, IEEE

Abstract—Banyan networks are attractive for serving as the optical switch architectures due to their nice properties of small depth and absolutely signal loss uniformity. Combining the horizontal expansion and vertical stacking of optical banyan networks is a general scheme for constructing banyan-based optical switching networks. The resulting horizontally expanded and vertically stacked optical banyan (HVOB) networks usually take either a high hardware cost or a large network depth to guarantee the nonblocking property. Blocking behavior analysis is an effective approach to studying network performance and finding a graceful compromise among hardware cost, network depth, and blocking probability; however, little has been done to analyze the blocking behavior of general HVOB networks. In this paper, we study the overall blocking behavior of general HVOB networks, where an upper bound on the blocking probability of a HVOB network is developed with respect to the number of planes (stacked copies) and the number of stages. The upper bound accurately depicts the overall blocking behavior of a HVOB network as verified by an extensive simulation study, and it agrees with the strictly nonblocking condition of the network. The derived upper bound is significant because it reveals the inherent relationship among blocking probability, network depth, and network hardware cost, so that a desirable tradeoff can be made among them. In particular, our bound gives network developers an effective tool to estimate the maximum blocking probability of a HVOB network, in which different routing strategies can be applied with a guaranteed performance in terms of blocking probability, hardware cost and network depth. Our upper bound model predicts some unobvious qualitative behaviors of HVOB networks, and it draws an important conclusion that a very low blocking probability (e.g., less than 0.001 percent) can be achieved in a HVOB network without introducing either a significantly high hardware cost or a large network depth.

Index Terms—Optical switching networks, banyan networks, blocking probability, horizontal expansion, vertical stacking.

1 INTRODUCTION

The Internet is experiencing an exponential growth in bandwidth demand from large numbers of users in multimedia applications and scientific computing, as well as in academic communities and the military. As a result of Wavelength Division Multiplexing (WDM) technology, the number of wavelengths per fiber has been increased to hundreds or more with each wavelength operating at rates of 10Gbps or higher [1]. Thus, the use of all-optical mesh networks based on WDM technology holds a great promise to meet the Internet’s ever increasing bandwidth demands, because the mesh-in-nature Internet backbones are considered more capacity-efficient and survivable.

It is expected that the traffic carried on tens of fibers at each node in a WDM mesh network will soon approach several terabits per second. Switching such a huge amount of traffic electronically becomes very challenging, due to both the high cost of optical-electronic-optical conversion and the high costs related to heat dissipation and space consumption. Although terabit capacity IP routers based on electronics are now starting to appear, there is still a serious mismatch between the transmission capacity of WDM (especially DWDM) fibers and the switching capability of electronic routers. Therefore, the adoption of all-optical switching networks in WDM networks has been an active research area for nearly two decades. Optical switching networks not only have the potential to steer network traffic at the speed of hundreds of terabits per second or higher [2], but they also can be more cost-effective than their electronic counterparts, even for applications requiring lower throughput. It is envisioned that in future WDM mesh networks, optical switching networks will play a key role in the transportation plane because they will be embedded with the intelligence of routing and signaling that enable them to handle complex mesh topologies and large numbers of inputs with different wavelengths, particularly at switching hubs that deal with a large volume of optical flows.

Directional coupler (DC) technology [3], [4] is promising for implementing the basic $2 \times 2$ switching elements (SEs) in an optical switching network. A DC is an electro-optical device implemented by manufacturing two waveguides close to each other. The cross (bar) state of a DC is created by applying a suitable voltage (or no voltage) to it. A DC can simultaneously switch optical flows with the speed of some terabits per second and with multiple wavelengths, so it is one of the best candidates to serve as the SE for future optical switching networks to support Optical Burst Switching and Optical Packet Switching.

To build a large-scale optical switching network based on basic SEs, numerous SEs are usually grouped in multiple stages along with the optical links, which are arranged in a specified interconnection topology between adjacent stages. The basic SEs and the interconnecting optical links will form a switching network such that the optical flows arriving at
Blocking behavior analysis of a network is an effective approach to the study of network performance and to finding a desirable trade-off between hardware cost and blocking probability. Lee [14] and Jacobaeus [15] have developed two well-known probabilistic models for analyzing the blocking behavior of Clos networks [16]. A number of studies with approaches similar to those proposed by Lee and Jacobaeus, have been conducted to analyze the performance of banyan networks [6], [7], [17], [18]; however, they present probabilistic results only for electronic networks. In other words, these studies only addressed link-blocking. Some analytical models have been developed to understand the blocking behaviors of vertically stacked optical banyan networks (without horizontal expansion) that do not meet the nonblocking condition (i.e., with fewer stacked copies than required by the nonblocking condition) [19], [20], [21]. To our best knowledge, however, no research has been reported for modeling and evaluating the performance behavior of general HVOB networks, in which not only the number of planes (network hardware cost) but also the number of stages (network depth) are incorporated in the performance analysis. Thus, this paper is committed to analyzing the overall blocking behavior of a HVOB network by deriving the upper bound on its blocking probability with respect to the number of planes and number of stages in the network. The main contributions of our work are the following:

- We extend the probabilistic methods used to analyze electronic networks, where only link-blocking is concerned, such that the performance in terms of overall blocking behavior in a general HVOB network can be analyzed, where the crosstalk-free constraint is the main contribution to blocking probability and both number of planes and number of stages are jointly considered.
- We propose an analytical model for evaluating the upper bound on blocking probability of a HVOB network. The bound demonstrates the inherent relationships among blocking probability, network depth and network hardware cost in terms of the number of vertically stacked planes in the HVOB network, and it is consistent with the strictly nonblocking condition of the network, so it can nicely describe the overall blocking behavior of the HVOB network in which different routing algorithms might be adopted.
- We develop a network simulator to verify our bound and conduct extensive simulations on it. It is shown that our model correctly estimates the upper bound of a HVOB network. Further simulation results based on both random routing and packing strategy indicate that the blocking probabilities of those two routing strategies are all nicely upper-bounded by our bound.

Our bound reveals some unobvious behaviors of general HVOB networks, and it gives network designers an effective tool to evaluate the maximum blocking probability and to initiate a graceful compromise among hardware cost, network depth and blocking probability in a HVOB network which may adopts different routing strategies.

The rest of the paper is organized as follows: Section 2 provides preliminaries that will facilitate the discussion. Section 3 introduces the proposed bound for a HVOB network. Section 4 presents the simulation results for model.
validation and also the performance discussions for HVOB networks. Section 5 concludes the paper.

2 PRELIMINARIES

A typical $N \times N$ banyan network has $\log N$ stages\(^1\) and one unique path between any input-output pair. One basic technique for creating multiple paths between an input-output pair is horizontal expansion, in which the reverse of the first $x(1 \leq x \leq \log N - 1)$ stages of a regular $N \times N$ banyan network is appended to the back of the network such that $2^x$ paths are created between the input-output pair, as illustrated in Fig. 2 for a $64 \times 64$ banyan network.

1. In this paper, log means the logarithm to the base 2.

Another technique for generating multiple paths between an input-output pair is the vertical stacking of multiple banyan networks [22]. The general scheme for building banyan-based optical switching networks is a combination of the horizontal expansion and vertical stacking of an optical banyan network [9], [10], as illustrated in Fig. 1. For simplicity, we use HVOB($N, m, x$) to denote an $N \times N$ HVOB network that has $m$ stacked planes of an $N \times N$ optical banyan network with $x$ extra stages.

The consideration of the crosstalk-free constraint distinguishes the analysis of optical switching networks from that of electronic ones. In electronic switching networks, blocking occurs when two connections intend to use the same link, which is referred to as link-blocking. Obviously, all signals passing through a network should follow link-disjoint paths.
in transmission to avoid link-blocking. In HVOB networks, however, we need to address another type of blocking. If adding the connection causes some paths, including the new one, to violate the crosstalk-free constraint, the connection cannot be added even if the path is available. We refer to this second type of blocking as crosstalk-blocking. Since the crosstalk-free constraint requires that no two optical signals ever share an SE in transmission (i.e., they should be node-disjoint in transmission), we need to consider only the crosstalk-blocking in HVOB networks. Obviously, the consideration of crosstalk-blocking will increase the overall blocking probability than considering only the link-blocking.

Due to their symmetric structures, all paths in banyan networks have the same property in terms of blocking. We define the blocking probability as the probability that a feasible connection request is blocked, where a feasible connection request is a connection request between an idle input port and an idle output port of a network. Without loss of generality, we choose the path between the first input port and the first output port (which is termed the tagged path in the following context) for the blocking analysis. All the SEs and links on the tagged path are called tagged SEs and tagged links, respectively. For a banyan network with \( x \) extra stages, we number the stages of SEs from left (stage 1) to right (stage \( \log N + x \)). We define the input intersecting set \( I_i = \{2^{i-1}, 2^i - 1, \ldots, 2^i - 1\} \) associated with stage \( i \) as the set of all inputs that intersect a tagged SE at stage \( i \). We define the output intersecting set \( O_i = \{2^{i-1}, 2^i - 1, \ldots, 2^i - 1\} \) associated with stage \( i \) as the set of all outputs that intersect a tagged SE at stage \( \log N + x - i + 1 \), as illustrated in Fig. 2.

To establish the upper bound on the blocking probability of a HVB network, we adopt a "conservative" routing control strategy [9] in bound derivation, in which all those connections that block a tagged path should fall within distinct planes to guarantee the nonblocking property such that the maximum number of blocked planes can be achieved based on the routing strategy. Here, we define a plane as a blocked plane if all its tagged paths are blocked. Thus, the connection request between the first input port (input 0) and the first output port (output 0) in a HVOB network will be blocked if all the planes of the network are blocked planes.

To simplify the analysis, we make the same assumption held in [14], [15] for multistage interconnection networks: The correlation between signals arriving (or leaving) at different input (or output) ports will be neglected. This leads to a fact that the status (either busy or idle) of each individual input (output) port in the network is independent. This assumption matches the practical situation since optical switching networks are becoming larger in size with increasingly complex interconnections, so as to transport a huge amount of data at once. In such circumstances, instead of being fixed with a certain extent of mutual correlation, the communication patterns of the input (or output) signals to an optical switch are becoming statistically random such that the correlation between signals at input (or output) ports becomes approximately negligible.

### 3 Upper Bound on Blocking Probability

We take \( NBP(N, x) \) to denote the number of blocked planes in a HVOB\((N, m, x)\) network under the "conservative" routing strategy. In this section, we first introduce the condition for the strictly nonblocking HVB\((N, m, x)\) network that is obtained by finding the maximum value of \( NBP(N, x) \), then we develop the upper bound on the blocking probability of a HVOB\((N, m, x)\) network for the cases of even and odd numbers of stages, respectively.

#### 3.1 Condition for Strictly Nonblocking

Let the maximum value of \( NBP(N, x) \) be \( \max \{NBP(N, x)\} \), then a HVOB\((N, m, x)\) network is strictly nonblocking if \( m \geq 1 + \max \{NBP(N, x)\} \) [9]. Thus, we only need to evaluate \( \max \{NBP(N, x)\} \) to determine the nonblocking condition. The maximum value of \( NBP(N, x) \) has been studied in [9]. Here, we study the maximum value of \( NBP(N, x) \) from a different perspective. The method will be used later to prove Theorem 1.

**Lemma 1.** The maximum value of \( NBP(N, x) \) is \( 2x + 2\sqrt{N}/2^x - 2 \) when \( \log N + x \) is even. When \( \log N + x \) is odd, the maximum value becomes \( 2x + (3/2)\sqrt{2N}/2^x - 2 \).

**Proof.** As shown in [9], [19], under the crosstalk-free constraint we can prove easily that:

\[
\max \{NBP(N, 0)\} =
\begin{cases} 
2\sqrt{N} - 2, & \text{if } \log N \text{ is even} \\
(3/2)\sqrt{2N} - 2, & \text{if } \log N \text{ is odd.}
\end{cases}
\]  

Here, we focus on the maximum value of \( NBP(N, x) \) when \( x \geq 1 \), and use \( B(N, x) \) to denote an \( N \times N \) banyan network with \( x(x \geq 1) \) extra stages. A \( B(N, x) \) network can be defined in a recursive way and this recursive definition will end at the central column of \( 2^x \) banyan networks \( B(N/(2^x), 0) \), as shown in Fig. 3.

**Fig. 3.** Recursive definition of \( B(N, x) \) network \( (x \geq 1) \). (a) The first step of the recursive definition. (b) The last step of the recursive definition. (a) \( B(N, x) \) and (b) \( B(N/2^{x-1}, 1) \).
Note that the $B(N, x)$ is just a plane of a HVOB($N, m, x$) network, and this plane is blocked if both its upper and lower tagged paths are blocked. Let $P_1(N)$ be the number of planes blocked only by the connections passing through the first and/or the last tagged SEs in a HVOB($N, m, x$) network under the “conservative” routing control strategy, then we have the following theorem.

The upper bound on the blocking probability of a connection request under any routing strategy, in which each event happens, we have the following theorem.

**Theorem 1.** For a HVOB($N, m, x$) network, the probability $Pr(NBP(N, x) = d)$ is given by:

$$Pr(NBP(N, x) = d) = \sum_{d_1, \ldots, d_N \leq d} Pr(P_1(N) = d_0).$$

$$Pr \left( NBP \left( \frac{N}{2}, x - 1 \right) = d_1 \right) \times 2 \left( 1 - \sum_{i=0}^{d_1-1} Pr \left( NBP \left( \frac{N}{2}, x - 1 \right) = i \right) \right) \tag{4}$$

$$Pr(NBP(N, x) = d) = \sum_{d_1 \leq d} Pr(P_1(N) = d_0)$$

$$Pr \left( \min \{ NBP_1 \left( \frac{N}{2}, x - 1 \right), NBP_2 \left( \frac{N}{2}, x - 1 \right) \} = d_1 \right),$$

where $NBP_1(N/2, x - 1)$ and $NBP_2(N/2, x - 1)$ are the numbers of planes blocked in the upper $B(N/2, x - 1)$ and lower $B(N/2, x - 1)$ of the $B(N, x)$ network, respectively, and, thus, $\min \{ NBP_1(N/2, x - 1), NBP_2(N/2, x - 1) \}$ is the maximum number of planes that the upper $B(N/2, x - 1)$ and lower $B(N/2, x - 1)$ combined can block.

Note that

$$Pr \left( \min \{ NBP_1 \left( \frac{N}{2}, x - 1 \right), NBP_2 \left( \frac{N}{2}, x - 1 \right) \} = d_1 \right) =$$

$$Pr \left( NBP_1 \left( \frac{N}{2}, x - 1 \right) = d_1, NBP_2 \left( \frac{N}{2}, x - 1 \right) \geq d_1 \right) + Pr \left( NBP_1 \left( \frac{N}{2}, x - 1 \right) \geq d_1, NBP_2 \left( \frac{N}{2}, x - 1 \right) = d_1 \right) - Pr \left( NBP_1 \left( \frac{N}{2}, x - 1 \right) = d_1, NBP_2 \left( \frac{N}{2}, x - 1 \right) = d_1 \right).$$

(6)

Based on the symmetric structure of the $B(N, x)$ network, we have:

$$Pr \left( NBP \left( \frac{N}{2}, x - 1 \right) = d_1, NBP_2 \left( \frac{N}{2}, x - 1 \right) \geq d_1 \right) =$$

$$Pr \left( NBP_1 \left( \frac{N}{2}, x - 1 \right) \geq d_1, NBP_2 \left( \frac{N}{2}, x - 1 \right) = d_1 \right)$$

$$Pr \left( NBP_1 \left( \frac{N}{2}, x - 1 \right) = d_1 \right) = Pr \left( NBP_2 \left( \frac{N}{2}, x - 1 \right) = d_1 \right)$$

$$= Pr \left( NBP \left( \frac{N}{2}, x - 1 \right) = d_1 \right) \tag{7}$$

and

$$Pr \left( NBP_1 \left( \frac{N}{2}, x - 1 \right) \geq d_1 \right) = Pr \left( NBP_2 \left( \frac{N}{2}, x - 1 \right) \geq d_1 \right) =$$

$$Pr \left( NBP \left( \frac{N}{2}, x - 1 \right) \geq d_1 \right). \tag{8}$$

(3)

$$1 - \sum_{d=0}^{2x+2\sqrt{N/2-2,m}+1} Pr(NBP(N, x) = d).$$

Since the “conservative” routing strategy, in which each of these connections that block a tagged path falls within a distinct plane, has been used in determining $NBP(N, x)$, the blocking probability of a connection request under any routing control strategy is then upper-bounded by the blocking probability in (3).

Equation (3) clearly indicates that we only need to evaluate the probability $Pr(NBP(N, x) = d)$ to get the upper bound on blocking probability. To calculate $Pr(NBP(N, x) = d)$, we shall establish the following theorem.

**Theorem 1.** For a HVOB($N, m, x$) network, the probability $Pr(NBP(N, x) = d)$ is given by:
Thus,

\[
\Pr(NBP(N/2, x - 1) = d_1, NBP(N/2, x - 1) = d_1) = \Pr(NBP(N/2, x - 1) = d_1) \cdot \Pr(NBP(N/2, x - 1) = d_1)
\]

\[
= \Pr(NBP(N/2, x - 1) = d_1) \cdot \Pr(NBP(N/2, x - 1) \geq d_1)
\]

To guarantee \( P_1(N) = 2 \) based on the “conservative” routing control strategy, both the second input port and the second output port must be occupied, and the connection passing through the second output port should not be destined for the second output port. Thus, the probability \( \Pr(P_1(N) = 2) \) is given by

\[
\Pr(P_1(N) = 2) = r \cdot r \cdot (N - 2)/(N - 1) = r^2 \cdot (N - 2)/(N - 1).
\]

Since \( P_1(N) \) can only take the values of 0, 1, and 2, we have

\[
\Pr(P_1(N) = 1) = 1 - \Pr(P_1(N) = 0) - \Pr(P_1(N) = 2) = 2r - r^2 \cdot (2N - 3)/(N - 1).
\]

This finishes our proof.

The evaluation of \( \Pr(NBP(N, 0) = d) \) is summarized in the following Lemma 3.

**Lemma 3.** For a HVOB \((N, m, 0)\) network, where \( \log N \) is even, the probability \( \Pr(NBP(N, 0) = d) \) is given by the following formula:

\[
\Pr(NBP(N, 0) = d) = \sum_{w_0 = 0}^{\min\{d, \sqrt{N - 1}\}} \sum_{w_O = d - w_0}^{\min\{d, \sqrt{N - 1}\}} f_1(w_1, w_1 + w_O - d) \cdot f_2(w_0, w_1 + w_O - d).
\]

(11)

Here, function \( f_1(x, y) \) is given by:

\[
f_1(x, y) = \sum_{i = 1}^{\min\{d, \sqrt{N - 1}\}} \sum_{T_i}^{\min\{d, \sqrt{N - 1}\}} \left( \prod_{i=1}^{\min\{d, \sqrt{N - 1}\}} \left( \frac{2^{i-1} - L_i}{L_i} \right)^{\alpha L_i} \cdot \beta_i^T \cdot (1 - \alpha - \beta_i)^{2^{i-1} - L_i - T_i} \right)
\]

where

\[
\alpha = r \times \left( \sqrt{N - 1}/(N - 1), \beta_i = r \times \left( N/2^{i-1} - \sqrt{N - 1}/(N - 1), i = 1, \ldots, (1/2) \log N, \right.ight.
\]

and \( r \) is the occupancy probability of an input (output) port.

Theorem 1 clearly shows a recursive relationship between \( \Pr(NBP(N, x) = d) \) and \( \Pr(NBP(N/2, x - 1) = d_1) \), and we will be able to use (4) to calculate the probability \( \Pr(NBP(N, x) = d) \) recursively if we can get the results for both probabilities \( \Pr(P_1(N) = d) \) and \( \Pr(NBP(N, 0) = d) \) (where \( \log N \) is even). For the probability \( \Pr(P_1(N) = d) \), we have the following Lemma 2.

**Lemma 2.** For a HVOB \((N, m, x)\) network, the probability \( \Pr(P_1(N) = d) \) is given by:

\[
\Pr(P_1(N) = d) = \begin{cases} 
(1 - r)^2, & \text{if } d = 0 \\
2r - r^2 \cdot (2N - 3)/(N - 1), & \text{if } d = 1 \\
r^2 \cdot (N - 2)/(N - 1), & \text{if } d = 2,
\end{cases}
\]

(10)

where \( r \) is the occupancy probability of an input (output) port.

**Proof.** Note that \( P_1(N) \) is the number of planes blocked only by the connections passing through the first and/or the last tagged SEs in a HVOB \((N, m, x)\) network, and we have at most two such kinds of connections (the connection passing through the second input port and the connection passing through the second output port of the network); therefore, the three possible values of the random variable \( P_1(N) \) are 0, 1, and 2.

Note that the event \( P_1(N) = 0 \) happens if and only if neither the second input port nor the second output port can be occupied. Let the occupancy probability of an input (output) port be \( r \), then we have

\[
\Pr(P_1(N) = 0) = (1 - r) \cdot (1 - r) = (1 - r)^2.
\]
of blocked planes will be \( w_I + w_O - k \). To guarantee \( NBP(N, x) = w_I + w_O - k = d \), we must have \( k = w_I + w_O - d \) connections from \( \bigcup_{i=1}^{(1/2)\log N} I_i \) that are destined for \( \bigcup_{i=1}^{(1/2)\log N} O_i \). Note that

\[
\left| \bigcup_{i=1}^{(1/2)\log N} O_i \right| = \left| \bigcup_{i=1}^{(1/2)\log N} O_i \right| = \sqrt{N} - 1
\]

and \( k = w_I + w_O - d \geq 0 \), we have:

\[
Pr(NBP(N, 0) = d) = \sum_{w_I=0}^{\min \{d, \sqrt{N} - 1\}} \sum_{w_O=d-w_I}^{\min \{d, \sqrt{N} - 1\}} Pr(k = w_I + w_O - d) \cdot Pr(w_I, w_O).
\]

Based on the treatments established in [19], we can then prove that the probability \( Pr(NBP(N, 0) = d) \) may be evaluated using (11)-(14).

Based on the results of Lemma 1, we can see easily that the upper bound of the blocking probability derived above matches the strictly nonblocking condition of a HVOB network [9], as summarized in the following corollary.

**Corollary 1.** For a HVOB \((N, m, x)\) network, where \( \log N + x \) is even, the blocking probability \( Pr^+(blocking) \) given in (3) becomes 0 if

\[
m \geq \max \{NBP(N, x)\} + 1 = 2x + 2\sqrt{N}/2^x - 1.
\]

### 3.3 Upper Bound on Blocking Probability When \( \log N + x \) Is Odd

Based on Lemma 1, the blocking probability \( Pr^+(blocking) \) of a HVOB \((N, m, x)\) network, where \( \log N + x \) is odd (please refer to Fig. 2b), is given by:

\[
Pr^+(blocking) = 1 - \sum_{d=0}^{\min \{2x + (3/2)\sqrt{N}/2^x - 2, m-1\}} Pr(NBP(N, x) = d).
\]

The probability \( Pr(NBP(N, x) = d) \) can be evaluated based on the recursive formula in (4), in which the probability \( Pr(P1(N) = d) \) is given by (10) and the evaluation of \( Pr(NBP(N, 0) = d) \) (where \( \log N \) is odd) is summarized in the following Lemma 4.

**Lemma 4.** For a HVOB \((N, m, 0)\) network, where \( \log N \) is odd, the probability \( Pr(NBP(N, 0) = d) \) is given by the following formula:

\[
Pr(NBP(N, 0) = d) = \sum_{w_I=0}^{\min \{d, \sqrt{N} - 1\}} \sum_{w_O=d-w_I}^{\min \{d, \sqrt{N} - 1\}} g_1(w_I, w_I + w_O - d) \cdot g_1(w_O, w_I + w_O - d) / g_2(w_I + w_O - d).
\]

Here, function \( g_1(x, y) \) is given by:

\[
g_1(x, y) = \sum_{T_i = 1}^{(1/2)\log N - 1} \prod_{i=1}^{(1/2)\log N - 1} \left( \begin{array}{c} 2i-1 \\log N \end{array} \right) \cdot \xi^{x_i} \cdot (1 - \xi)^{y_i} \cdot (1 - \xi)^{w_i}.
\]

And function \( g_2(x) \) is given by:

\[
g_2(x) = \left( \sqrt{2N - 1} \right)^x \cdot (1 - \xi)^{\sqrt{2N - 1} - x}.
\]

where

\[
\xi = r \times \left( \sqrt{2N - 1} \right)/(N - 1),
\]

\[
\eta_i = r \times \left( N/2^{i-1} - \sqrt{2N} \right)/(N - 1),
\]

\( i = 1, \cdots, (1/2)(\log N - 1) \).

**Proof.** The lemma can also be proven based on the treatments established in [19].

The following corollary indicates that, when \( \log N + x \) is odd, the upper bound blocking probability we derived also matches the condition for a strictly nonblocking HVOB network [9].

**Corollary 2.** For an HVOB \((N, m, x)\) network, where \( \log N + x \) is odd, the blocking probability \( Pr^+(blocking) \) given in (15) becomes 0 if

\[
m \geq \max \{NBP(N, x)\} + 1 = 2x + (3/2)\sqrt{N}/2^x - 1.
\]

### 4 Experimental Results and Discussions

An extensive simulation study has been conducted to verify our upper bound on the blocking probability (also denoted by \( BP \) hereafter) of a HVOB network. Our network simulator consists of the following two modules: the request pattern generator and request router. The request pattern generator randomly generates a set of connection request patterns for a HVOB network based on the occupancy probability \( r \) of an input/output port. To verify the upper bound on \( BP \), the “conservative” routing strategy, random routing strategy, and packing strategy [23] are used in the request router to route the connection requests in a connection pattern through the HVOB network. In the “conservative” routing strategy, each connection request has the probability of 0.5 to go through either the upper or the lower part of the network recursively, and we guarantee that all the requests that block a specified tagged path will fall within distinct plans. To establish the connection request in random routing, the request router randomly chooses one of the planes that can be used by a request to establish the connection. Under the
packing strategy for a HVOB network, a connection is realized on a path found by trying the most used plane of the network first and the least used plane last. In a HVOB \(\text{HVOB}(N, m, x)\) network, a plane is blocked if all its tagged paths are blocked. For a connection pattern, if no plane can satisfy the request of the tagged path using a routing strategy, the connection pattern is recorded as a blocked connection pattern corresponding to the routing strategy. The blocking probability of a routing strategy is then estimated by the ratio of the number of blocked connection patterns to the total number of connection patterns generated. During the simulation, a certain workload is maintained. The workload is measured by the network utilization, which is defined as the probability that an input (output) port is busy.

4.1 Theoretical versus Simulated Upper Bounds on BP

We have examined two networks, HVOB(512, \(m, x\)) and HVOB(1024, \(m, x\)) with \(x = \{1, 2\}\), to verify the derived upper bound. For each network configuration, blocking probability is examined by using both the theoretical bound and the simulator for \(r = 0.8\). The corresponding results are summarized in Fig. 4 and Fig. 5.

The results in Fig. 4 and Fig. 5 show clearly that our theoretical model correctly estimates the upper bound on the blocking probability of general HVOB networks, and the results from the random routing and packing strategy are all nicely bounded by the derived upper bound. It is notable that the theoretical upper bound follows closely the condition of a strictly nonblocking HVOB network [9]. For the network with \(N = 512\), the upper bound goes to zero at \(m = 2\sqrt{N/2} + 1 = 33\) when the network has one extra stage and goes to zero at \(m = (3/2)\sqrt{2N/4} + 3 = 27\) when the network has two extra stages. For HVOB(1024, \(m, 1\)) network and HVOB(1024, \(m, 2\)) network, the upper bound of blocking probability becomes zero at \(m = (3/2)\sqrt{2N/2} + 1 = 49\) and \(m = 2\sqrt{N/4} + 3 = 35\), respectively. The results in Fig. 4 and Fig. 5 also indicate that for a given network configuration, it is possible for us to dramatically reduce the number of planes by tolerating a predictable and negligibly small blocking probability.

4.2 Network Depth versus BP and Hardware Cost

To show the impact of increasing network depth upon blocking probability, we illustrate in Fig. 6 the blocking probabilities of different HVOB(\(N, m, x\)) configurations with \(N = \{512, 1024\}\) and \(x = \{0, 1, 2, 3\}\) at the network utilization of \(r = 0.9\). We observe from Fig. 6 that for the two networks we studied, given a constant number of planes and a constant network utilization, the blocking probability decreases sharply as the number of extra stages increases from 0 to 2, but this decrease in blocking probability becomes insignificant if we increase number of extra stages further from 2 to 3. The results in Fig. 6 also indicate clearly that for a given constraint on blocking probability in a HVOB(\(N, m, x\)) network, we can reduce the
required number of planes by increasing the number of stages in the network, and there exists a tradeoff between these two parameters. To show this tradeoff clearly, Fig. 7 illustrates the hardware cost (total number of required SEs) versus the number of extra stages for different HVOB networks with $BP < 0.1\%$ and $BP < 1\%$ at the network utilization of $r = 0.9$ (Fig. 7b).

Although we can always reduce the number of planes by increasing the number of stages for a given constraint on blocking probability, the results in Fig. 7 indicate that an optimal trade-off in terms of the total number of required SEs can be achieved by appending only a small number (two, here) of extra stages to a HVOB network. Thus, our model can guide network designers to find the optimal HVOB structure for an optical switching network with a specified $BP$ requirement. Fig. 7 actually reveals an unobvious overall behavior of HVOB networks, that for a given $BP$ requirement, we can achieve a least cost by appending only a small number of extra stages to the networks. Note that a small network depth is always preferred because crosstalk and signal attenuation are proportional to the number of couplers that a light signal passes through, so the above attractive behavior makes HVOB networks promising in practical applications. Interestingly, the results in Fig. 7 further indicate that although the total number of required SEs varies with the variations in workload and the requirement of $BP$, the optimal HVOB structure is robust in the sense it is not sensitive to variations of these two parameters.

4.3 Hardware Cost versus BP and Workload

Fig. 7 indicates that for different HVOB networks we studied, an optimal trade-off in terms of total number of required SEs can be achieved by appending only two extra stages. To find out more about the sensitivity of hardware cost to the requirements of $BP$ and workload, we focus on the HVOB($N, m, 2$) architecture and show in Table 1 the minimum number of planes estimated by our upper bound for different $BP$ requirements and different workloads. For comparison, we also show in Table 1 the minimum number of planes determined by the nonblocking condition of a strictly nonblocking HVOB network with $BP = 0$.

The results in Table 1 indicate that, for larger HVOB($N, m, 2$) networks, the hardware costs for the nonblocking condition are considerably higher than those given by the proposed upper bound, even under a strict constraint on blocking probability. For the $1.024 \times 1.024$ HVOB network with two extra stages, the minimum number of planes determined by the nonblocking condition is 35 while the minimum number of planes given by our bound is only 15 for $BP < 0.001\%$ and $r = 1.0$. The above implies that $(35 - 15)/35 \approx 57\%$ of the hardware cost can be reduced while a very low blocking probability is guaranteed ($BP < 0.001\%$). It is also interesting to observe from both Table 1 and Fig. 7 that compared to the variation of $BP$ requirement, the hardware cost estimated by our upper bound is more sensitive to the variation of workload $r$. For the HVOB($512, m, 2$) network with $r = 1.0$, the minimum number of planes estimated by the upper bound is 12 for...
the requirement $BP < 1\%$; this number increases slightly to 13 when the requirement on $BP$ becomes $BP < 0.1\%$ (10 times stricter), and all the results are much less than the 27 planes required by the nonblocking condition. Again, for the HVOB($512, m, 2$) network, we need 11 planes to guarantee $BP < 0.1\%$ when workload is 0.75, but we require 14 planes to guarantee the same $BP$ requirement when the workload increases to 1.0 (only 25 percent higher), and still all these results are much less than the 27 planes required by the nonblocking condition.

Ideally, routing and switch controlling in an optical switching network should be performed all in optical domain to achieve a much higher capability than can be expected from an electronic switching network. However, these functions are very difficult to perform optically due to the very limited processing capabilities in the optical domain. One important factor is the huge technology barrier in practically implementing optical random access memory for buffering. Thus, to implement a HVOB network in real environments, the routing and path setup for a request should be processed electronically in a central controller such that the contention can be resolved before the transmission of optical data. How to select a path through the network for a request depends on what kind of routing strategies to be employed, such as, random routing, packing, etc. Our bound enables the network designers to estimate the maximum blocking probability of a HVOB network, in which different routing strategies may be applied. This analytical model can also help network designers to find the optimal HVOB structure for building an optical switching network with a specified constraint on blocking probability. Our model reveals an unobvious overall behavior of HVOB networks; the hardware cost of a HVOB network can be reduced dramatically while a small network depth and a negligible small blocking probability are guaranteed. We expect that the modeling method employed in this paper will help in deriving the upper bound on the blocking probabilities of other types of optical switching networks as well.

<table>
<thead>
<tr>
<th>$BP$</th>
<th>$N=64$</th>
<th>$N=128$</th>
<th>$N=256$</th>
<th>$N=512$</th>
<th>$N=1024$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$BP=0$</td>
<td>$r=1.0$</td>
<td>11</td>
<td>15</td>
<td>19</td>
<td>27</td>
</tr>
<tr>
<td>$BP&lt;0.001%$</td>
<td>$r=0.5$</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
</tr>
<tr>
<td>$BP&lt;0.01%$</td>
<td>$r=0.75$</td>
<td>8</td>
<td>10</td>
<td>10</td>
<td>12</td>
</tr>
<tr>
<td>$BP&lt;0.1%$</td>
<td>$r=1.0$</td>
<td>9</td>
<td>11</td>
<td>12</td>
<td>14</td>
</tr>
<tr>
<td>$BP&lt;1%$</td>
<td>$r=0.5$</td>
<td>5</td>
<td>6</td>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td>$BP&lt;5%$</td>
<td>$r=0.75$</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>10</td>
</tr>
</tbody>
</table>

5 CONCLUSIONS

In this paper, we have developed an analytical model for evaluating the upper bound on the blocking probability of general HVOB networks that employ both horizontal expansion and vertical stacking of banyan networks. Extensive simulation results prove that the derived bound, which agrees with the strictly nonblocking condition of a HVOB network, accurately depicts the overall blocking behavior of the HVOB network. The model provides network developers with guidance for quantitatively determining the impact of appending extra stages and reducing the number of planes on the overall blocking behavior of a HVOB network in which different routing strategies may be applied. This analytical model can also help network designers to find the optimal HVOB structure for building an optical switching network with a specified constraint on blocking probability. Our model reveals an unobvious overall behavior of HVOB networks; the hardware cost of a HVOB network can be reduced dramatically while a small network depth and a negligible small blocking probability are guaranteed. We expect that the modeling method employed in this paper will help in deriving the upper bound on the blocking probabilities of other types of optical switching networks as well.

ACKNOWLEDGMENTS

This research is partly supported by the Grand-In-Aid of scientific research (B) 14380138 and 16700056, Japan Science Promotion Society. The authors would like to thank the anonymous reviewers for their valuable and constructive comments.

REFERENCES

Chen Yu received the BSc and MEng degrees in 1998 and 2002, from Wuhan University, Wuhan, China. He is now a doctoral student in the Department of Computer Science, Graduate School of Information Science, TOHOKU University, Japan. He was the research assistant and research supporter in the Graduate School of Information Science, Japan Advanced Institute of Science and Technology (JAIST), from September 2002 to March 2004. Now, he is also the research assistant and teaching assistant in the Graduate School of Information Science, TOHOKU University. His research interests include interconnection networks, optical switch networks, WDM networks, security on Internet and data mining.

Xiaohong Jiang received the BS, MS, and PhD degrees in 1988, 1992, and 1999, respectively, all from Xidian University, Xi’an, China. He is currently an associate professor in the Department of Computer Science, Graduate School of Information Science, TOHOKU University, Japan. Before joining TOHOKU University, he was an assistant professor in the Graduate School of Information Science, Japan Advanced Institute of Science and Technology (JAIST), from October 2001 to January 2005. Dr. Jiang was a JSPS (Japan Society for the Promotion of Science) postdoctoral research fellow at JAIST from October 1999 to October 2001. He was a research associate in the Department of Electronics and Electrical Engineering, the University of Edinburgh from March 1999 to October 1999. His research interests include optical switching networks, WDM networks, interconnection networks, IC yield modeling, timing analysis of digital circuits, clock distribution, and fault-tolerant technologies for VLSI/WSI. He has published more than 60 referred technical papers in these areas. He is member of the IEEE.

Susumu Horiguchi received the BEng, MEng, and PhD degrees from Tohoku University in 1976, 1978, and 1981, respectively. He is currently a professor and Chair of Department of Computer Science, the Graduate School of Information Science, Chair of Department of Information Engineering, Faculty of Engineering, Tohoku University. He was a visiting scientist at the IBM T.J. Watson Research Center from 1986 to 1987. He was also a professor in the Graduate School of Information Science, JAIST (Japan Advanced Institute of Science and Technology). He has been involved in organizing international workshops, symposia, and conferences sponsored by the IEEE, IEICE, IASTED, and IPS. He has published more than 150 papers technical papers on optical networks, interconnection networks, parallel algorithms, high performance computer architectures, and VLSI/WSI architectures. He is a senior member of the IEEE and member of IPS and IASTED.

Minyi Guo received the PhD degree in computer science from University of Tsukuba, Japan in 1998. From 1998 to 2000, Dr. Guo had been a research scientist of NEC Soft, Ltd. Japan. He is currently a full professor in the Department of Computer Software, The University of Aizu, Japan. From 2001 to 2004, he was a visiting professor of Georgia State University, Hong Kong Polytechnic University, and University of New South Wales, Australia. He has published more than 100 papers in major journals and refereed conference proceedings related to the research areas. Dr. Guo has served as general chair, program committee, or organizing committee chair for many international conferences. He is the editor-in-chief of the International Journal of Embedded Systems. He is also in editorial board of International Journal of High Performance Computing and Networking, the Journal of Embedded Computing, the Journal of Parallel and Distributed Scientific and Engineering Computing, and the International Journal of Computer and Applications. His research interests include parallel and distributed processing, parallelizing compilers, data parallel languages, data mining, molecular computing, and software engineering. He is a member of the ACM, IEEE, IEEE Computer Society, IPSJ, and IEICE.